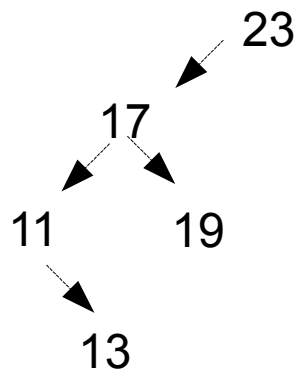


## Instructions:

Open book, notes, Internet  
Do NOT contact another human (except professor)  
The final can be submitted via  
email,  
web (give URL),  
paper (slip under my door 511E Pray Harrold)  
The final can be submitted with multiple files.

1. Consider this splay tree. Each of the following two questions starts with this splay tree.



1. a. What is the splay tree obtained from accessing node 11?

1. b. What is the splay tree obtained from accessing node 13?

2. Build the kD tree,  $k=2$ , for the following data.

Insert the data to an initially empty 2D tree in the order given:

2 a.  $(10, 10)$  ,  $(50, 90)$  ,  $(100, 50)$  ,  $(90, 60)$  ,  $(120, 40)$  ,  $(80, 70)$  ,  $(95, 55)$  .

2 b. The 2D tree you built to answer 2 a is very out of balance. Propose an algorithm that will build a more balanced 2D tree. (A large Big Oh is not relevant to this question). Then, give the 2D tree that results from inserting the above data using your proposed algorithm.

3. Consider this alphabet with the measured frequencies as shown.

char	freq
a	.65
b	.20
c	.10
d	.03
e	.02

3 a. Give a Huffman code for this alphabet (the Huffman code can be in tree form or in table lookup form).

3 b. Encode the message “d d d e e e” ( spaces inserted for readability ).

3 c. What is the compression obtained for your encoding of “d d d e e e”?

4. Find a minimum spanning tree for this undirected graph of six vertices.

- An empty cell indicates no edge between the two vertices.
- Start with vertex v.
- State the algorithm you are using.
- Show the intermediate solutions obtained after each vertex is added to the MST.

	v	w	x	y	z	a
v	0	4	2	1	2	
w	4	0	1	1		
x	2	1	0	3		1
y	1	1	3	0	2	3
z	2			2	0	
a			1	3		0

xxxx. Compute the Fourier transform for (2 -1 1 1). Show the steps after the fashion shown in <https://www.youtube.com/watch?>

[v=KxIjfaeOD5U&feature=BFa&list=PL4EFC49B5C43AD082&lf=plpp\\_video](https://www.youtube.com/watch?v=KxIjfaeOD5U&feature=BFa&list=PL4EFC49B5C43AD082&lf=plpp_video)

where the recursion is traced:  $a[]$  and  $y[]$  are explicitly given for  $n=4, 2, 1, 0$ .

5. a. Show the dynamic programming results for the strings “drink” and “drank”. The cost for inserting a blank (  $\delta$  ) is 2. The cost for any character mismatch (  $\alpha$  ) is 1.

5. b. From your results in 5. a., give the best alignment(s).

6. Consider the directed graph with demands and lower bounds. Give a feasible solution. The capacity of each edge is entered to the appropriate cell in the matrix as (le, ce) for (lower bound, upper bound). Note, if  $le = 0$ , then there is no lower bound constraint on the flow.

demand		a	b	c	d	e
-3	a		0, 3	0, 3	0, 1	
0	b			1, 1	1, 2	
0	c		0, 1			1, 2
1	d					0, 2
2	e					

To show your work (for partial credit if needed), give the augmenting path at each stage.