SOUNDOFF

MATHEMATICS: A WONDERFUL KIND OF PLAY

By CRAIG B. MEROW, Delaware Valley School District, Milford, PA 18337

When Gerald L. Alexanderson interviewed Constance Reid for the book *Mathematical People* (Albers and Alexanderson 1985), he asked her what it was about mathematics and mathematicians that originally excited her.

"That's easy," she replied, "mathematics is a world created by the mind of man, and mathematicians are people who devote their lives to what seems to me a wonderful kind of play!"

Regrettably, the majority of high school students I have encountered would not describe mathematics in this way. Although these students may have mastered the mechanics of the quadratic formula and committed the proper definition of adjacent angles to memory, they have not been exposed to the vibrant, ongoing quest we call "mathematics." Their mathematics courses have tended to follow the text very closely. Few have been asked to investigate recent developments in mathematics, to interview mathematicians, to write papers, or to work on group projects. As a result, most high school students are surprised to learn that new theorems are being written every day, and few can name a living mathematician.

During the past ten years I have been the mathematics department chair at Germantown Academy and Delaware Valley High School. In this capacity I have interviewed over one hundred candidates for mathematics teaching positions. During these interviews I attempt to afford candidates an opportunity to demonstrate an understanding of the spirit of mathematics, to discuss their involvement in the mathematical community, and enthusiastically to express their desire to draw students into the wider world of mathematics. I look for a twinkle in their eyes as they discuss mathematics—the same twinkle I know Constance Reid had when she described mathematics as "a wonderful kind of play!"

I am sad to say that the "twinkle" is often lacking. The following is a typical interview.

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Merow: Tell me about your academic interests.

Candidate: Academic interests?

Merow: What are you interested in? What subjects did you enjoy in college? What do you read?

Candidate: Well, I like math.

Merow: Do you belong to any mathematics organizations, such as the National Council of Teachers of Mathematics, the Mathematical Association of America, or the Consortium for Mathematics and Its Applications?

Candidate: No.

Merow: Do you regularly read any mathematics periodicals?

Candidate: I looked at a couple of articles in the *Mathematics Teacher* for a paper for my methods class.

Merow: Tell me about them.

Candidate: It's been a long time. I don't really remember them very well. I think they were about primes or something.

Merow: Have you read any math books other than those required for your classes?

Candidate: What do you mean?

Merow: Have you read any of the mathematics books written for the educated layman, such as *The Mathematical Experience* (Davis and Hersh 1981), *Bridges to Infinity* (Guillen 1983), *Godel, Escher, Bach* (Hofstadter 1979), and *Mathematical People* (Albers and Alexanderson 1985)?

Candidate: No.

Merow: Flatland (Abbott 1984)?

Candidate: No. I've heard of it. I think I know someone who read it.

Merow: Tell me something about your taste in mathematics.

Candidate: Taste in mathematics?

Merow: Are you more interested in theoretical mathematics or in applications? What courses particularly interested you in college?

Candidate: I liked calculus.

Merow: Why?

Candidate: I did well in it.

Merow: What is your favorite theorem? What is the mathematical idea that got you excited, that convinced you that you wanted to study mathematics?

Candidate: Well, I don't remember many theorems. There is the Pythagorean theorem.

Merow: Is there anyone in particular whose work you admire?

Candidate: Do you mean a mathematician?

Merow: Certainly!

Candidate: Like Euclid or somebody?

Merow: I was hoping that you were familiar with the work of some contemporary mathematicians. Can you name a living mathematician?

Candidate: (Silence)

Merow: Does the name Gerd Faltings mean anything to you? Benoit Mandelbrot? Paul Erdos?

Candidate: (Shakes his or her head "no")

Merow: How about historical figures?

Candidate: (Looks puzzled)

Merow: Dead ones!

Candidate: Einstein was a good mathematician. I've always found relativity interesting—time slowing down, things like that.

The mathematics teachers who are "interested in mathematics" but do not participate in any way in the mathematical community often express surprise at my questions. They make remarks such as, "I expected you to ask me how I would introduce negative numbers or if I would feel comfortable teaching geometry." Almost all expect the discussion to be limited to the content of high school mathematics and to their methods of "covering" these topics.

High school administrators have also expressed puzzlement over my line of questioning. "Is that a fair question?" I was asked by an assistant principal after asking a candidate if she could name a living mathematician. "Would you hire an English teacher who couldn't name a living novelist? A basketball coach who couldn't name a living basketball player?" I replied.

I was taught the importance of vicarious experience and the role of the hero in the development of children's values by a neigh-

borhood boy with a passion for basketball. Every day after school he would shoot baskets until his mother forced him to come in for dinner. "If only my students were devoted to mathematics the way that young man is to basketball," I often thought.

As the boy practiced week after week, his dedication really began to intrigue me. "What is so interesting about trying to throw a ball through a hoop over and over again?" I wondered. Even a routine problem set should be as exciting.

I began to watch the boy more closely. I noticed that he would often shout the name of a favorite player as he went up for a shot. And after a successful shot he would exclaim, "Two points for Kirchoff!" My little friend was not simply trying to throw a ball through a dilapidated hoop attached to his garage; he was participating in something much grander. He had been to professional basketball games. He had heard the screams of a crowd; he had felt the tension as the final minutes of an important game ticked away; he had been awed by the herculean efforts of a star who could turn a game around in the final moments. In shooting baskets he was becoming a part of something full of drama and excitement.

The mathematical community's approach to training a new generation of mathematicians is to have them run drills for sixteen years before they get to see the game. Students learn to recite the multiplication tables, to factor polynomials, to differentiate trigonometric functions, and to orthogonalize a basis for $\mathbf{R}^4$. And after sixteen years of mathematics education most cannot name a living mathematician. They have no mathematical heroes. They do not know what takes place in the mathematics department at Bell Laboratories, about Faltings's proof of Mordell's conjecture, or about the excitement and disappointment over the recent attempts to prove Fermat's "last theorem." Although they have acquired some technical skills, they have not been made to feel a part of something larger.

The mathematical equivalent of a tournament-winning goal is a mental event hidden from the view of others. Vicarious experience in mathematics does not come as easily as in basketball, but it is possible.

Last year I shared Paul Hoffman's entertaining article about Paul Erdos, "The Man Who Loves Only Numbers" (1987), with one of my precalculus classes. Many of my students were impressed with Erdos's mental power and captivated by his playful use of language. They began to refer to younger students as "epsilons," Erdos's word for children, and often spoke reverently of the "Book." The "Book" is a book that Erdos likes to imagine that contains the most elegant proofs of all mathematical theorems and is seen in its entirety only by God.

A few weeks later a student proved a rather difficult theorem on the chalkboard. His proof was very clever. "That's straight from the Book!" I exclaimed. The boy beamed, just as my neighborhood basketball player would if he were compared to Michael Jordan. My student had been made to feel a part of the mathematical community.

After investigating the implications of modifying Euclid's axioms, John Bolyai is reported to have said, "I have made such wonderful discoveries that I am myself lost in astonishment: Out of nothing I have created a new and another world" (Kline 1953, 410). We must transmit this very excitement to students. What's more, we have to do it before they are capable of understanding Bolyai's work in detail.

The resources for this task exist in the expository literature. Few high school students have the mathematical sophistication to investigate Bolyai's geometry, but most can identify with Square's experience of being plucked from two-space into a three-dimensional universe by the Sphere in Edwin Abbott's fantasy, *Flatland* (1984). And through reading and discussing this book they can begin to see that questioning the geometry we take for granted is sensible and that investigating other possibilities could be exciting. *Flatland* should be required reading for all high school mathematics students.

I suggest that we include books about mathematics and mathematicians in all our courses, both at the high school and college levels; that we use videos, such as "For All Practical Purposes" (1987); that we invite working mathematicians to speak to our classes; and that we assign papers requiring investigation of the world of mathematics beyond the narrow confines of the textbooks.

To those who respond that they do not have the time, that they cannot even finish the present curriculum in the allotted time, I say "nonsense!" We are graduating high school seniors who cannot find the area of a triangle, not because they lack the ability to perform this mathematical feat or because we didn't "cover" the topic, but because of lack of motivation. To improve the mathematical achievement of America's youth we must show them that men and women can do exciting things with their minds and that they are greatly respected for their achievements. And most important, we must convince them that to varying extents they can become like these heroes. Whatever time is used to accomplish this goal will more than pay for itself with increased motivation. Students can learn mathematics by doing it, but they are motivated to learn by reading about it. Let's expand the scope of our offerings to students; let's show them the game—that "wonderful kind of play!"

BIBLIOGRAPHY

Abbott, Edwin A. *Flatland.* New York: New American Library, 1984.

Albers, Donald J., and G. L. Alexanderson, eds. *Mathematical People: Profiles and Interviews.* Cambridge, Mass.: Birkhäuser Boston, 1985.

Begley, Sharon, and David Grant. "A Triumph of the Intellect." *Newsweek,* 1 August 1983, 66.

Campbell, Douglas M., and John C. Higgins, eds. *Mathematics: People, Problems, Results.* Volume 2. Belmont, Calif.: Wadsworth, 1984, 73.

Cipra, B. A. "Doubts Over Fermat Proof." *Science,* 25 March 1988, 1481.

———. "Fermat's Theorem Proved?" *Science,* 18 March 1988, 1373.

———. "Fermat's Last Theorem Remains Unproved." *Science,* 3 June 1988, 1275–76.

Davis, Philip J., and Reuben Hersh. *The Mathematical Experience.* Cambridge, Mass.: Birkhäuser Boston, 1981.

Esbenshade, Donald H., Jr. "Adding Dimension to *Flatland:* A Novel Approach to Geometry." *Mathematics Teacher* 76 (February 1983):120–23.

For All Practical Purposes. Santa Barbara, Calif.: The Annenberg/CPB Project, 1987. Video.

Gleick, James. *Chaos.* New York: Viking Penguin, 1987.

Guillen, Michael. *Bridges to Infinity.* Los Angeles, Calif.: Jeremy P. Tarcher, 1983.

Hoffman, Paul. "The Man Who Loves Only Numbers." *Atlantic,* November 1987, 60–74.

Hofstadter, Douglas R. *Godel, Escher, Bach.* New York: Random House, 1979.

Kline, Morris. *Mathematics in Western Culture.* New York: Oxford University Press, 1953.

Kolata, G. "Number Theory Problem Is Solved." *Science,* 22 July 1983, 349–50.

Krauthammer, Charles. "The Joy of Math, or Fermat's Revenge." *Time,* 18 April 1988, 92.

Merow, Craig B. "Encounter with a Mathematician." *Mathematics Teacher* 80 (September 1987):471–72.

Peterson, I. "Fermat's Last Theorem: A Promising Approach." *Science News,* 19 March 1988, 180–81.

———. "Doubts about Fermat Solution." *Science News,* 9 April 1988, 230.

Sacket, P. D. "New Proof Finally Handles Old Math Problem." *Science News,* 23 July 1983, 58.

Tierney, John. "Paul Erdos Is in Town. His Brain Is Open." *Science,* October 1984, 40–47. ■