Investigation of supply disruption with time dependent parameters

Rong, Ying

December 19, 2005

Although supply disruption sounds very rare, it happens all the time. Besides those factors, such as weather, labor strike, terrorism, when the manufacturer production scale is very large, such as shampoo produced by P&G, those small retailer may not get their order at all due to randomness of manufacture and their small scale. Song and Zipkin [2] models this situation as discrete queueing system. Since those small retailer cannot get information of manufacture process whether it occurs disruption or manufacturer randomness, we can approximate the whole system as a disruption model.

The most research done so far assumes constant disruption rate and recover rate. Under constant disruption rate assumption, people like Parlar and Perry [1] studies how it affects those classic inventory policies, such as (r, Q) policy. Synder [3] provides a tight approximation for the lost sales EOQ model with disruption(EOQD). Song and Zipkin [2] study the case where supply system can be modelled as a discrete time Markov chain. After reducing state space, they shows the value of knowing information of supply system. Recently Tomlin and Synder study the supply system with multiple fail rates following discrete time Markov chain. They shows the state dependent order up level will gain significant saving again constant order up level policy.

In this paper, we consider the availability of one single supplier following continuous time Markov chain and one retailer facing Poisson demand. The events, such as adverse weather, strike, machine breakdown, congestion of orders from all retailers the supplier encounters, can cause the supplier incapable to provide its product to one or more retailers. Usually the probabilities of having those events as well as the Poisson-distributed demand received by the retailer are time dependent. We model this problem as two dimension CTMC and solve Kolmogorov CTMC differential equations numerically to acquire total cost under certain ordering policy. We propose several control forms of ordering policy for the retailer, which can be characterized into two categories. One is using time independent order quantity, and the other is real-time order quantity varying by the time.

We compare the those proposed policies under different cost, demand, disruption parameters by extensive computational experiments. The benefit of real-time order policy is investigate. At the same time, disruption is a low probability event. It is not easy to get accurate estimation of those time dependent parameters. The robustness of real-time policy is examined, and we find the real-time policy can balance optimality and robustness well if the parameters of control is set properly.

In section 1, we provide problem formulation and proposed policies. In section 2, the extensive of computational experiments on the benefit of real time order policy is presented. In section 3 , the relationship of parameters of optimal solution under certain control policy is studied. In section 4, the robustness investigation of policies is carried out. In section 5, we give the suggestion when to use real time ordering control policy.

1 Problem formulation and proposed policies

We suppose the supplier has two states. One is 'up' and the other is 'down'. When supplier is at 'up' state, it has ample inventory for its retailers. Otherwise the supplier has nothing to supply until its state switch from 'down' to 'up'. When the supplier is at 'up'/'down', the time to switch to 'down'/'up' is exponentially distributed. The demand faced by the retailer is Poisson distributed. The retailer uses the zero inventory ordering policy. The leadtime is 0 condition on 'up' state of the supplier. When the supplier is at 'down' state, the retailer will place its order right after the supplier recovers. Unsatisfied demand is lost. Based on these assumptions, Figure 1 shows CTMC diagram for the problem. The number in the figure denotes the inventory level at retailer. 'U'/'D' denotes 'up'/'down' state at supplier. Since the leadtime is 0 condition on 'U', $(0, U)$ cannot be achieved unless $Q = 0$.

We use the cosine form to approximate time dependent fail rate and demand rate. It is easy to make cost parameters and repair rate time dependent. But it is reasonable and convenient to fix it constant over time.

In order to demonstrate the problem formulation, we introduce the following notation[1].

[1] K, h, p, f_r can be time varying without any difficulty. But we let them constant over time in our experiments.

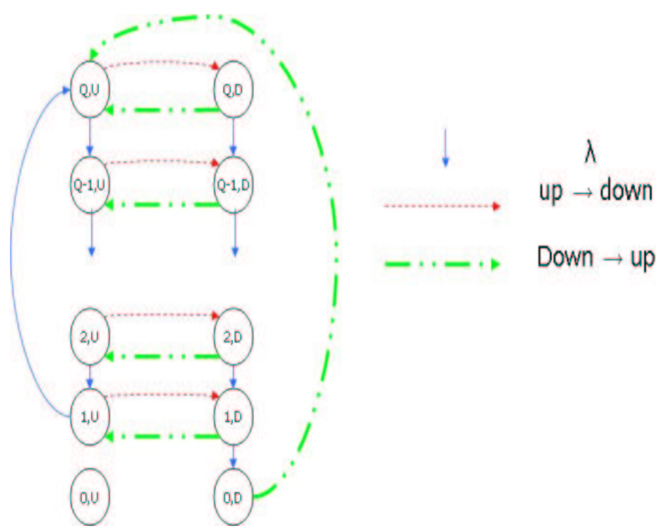

Figure 1: CTMC diagram

Parameters	Definition	Basic setting
K	Fixed cost	31
h	Holding cost	1
p	Stockout cost	11
f_r	Repair rate	12
$\overline{f_b}$	Average fail rate	1
fRA	Relative amplitude of fail rate	0.9
fPH	Phase of fail rate	0
$\overline{\lambda}$	Average demand rate	100
dRA	Relative amplitude of demand rate	0
dPH	Phase of demand rate	0

$\lambda(t) = \overline{\lambda}(1 - dRA\cos(2\pi(t + dPH)))$ Demand rate function

$IL(t)$ Inventory level. It is a random variable

$\delta(x)$: $\lim_{\Delta t \to 0} \int_t^{t+\Delta t} \delta(x)dx = 1$ when $t > 0$ and 0 otherwise

$Q(t)$: Our decision variable, real time order amount when inventory level hits zero.

Our expected long run average total cost is equal to,

$$C(T) = E[\int_0^T \{h[IL(t)]^+ + p[1 - IL(t)]^+\lambda(t) + K(\Pr(1,'U',t)\lambda(t) + \Pr(0,'D',t)f_b(t))dt]$$

where

$$\Pr(IL(t)) = \Pr(IL,'U') + \Pr(IL,'D')$$

If we are given a specific form of $Q(t)$ and corresponding parameters, we can solve Kolmogorov CTMC differential equations numerically to get distribution of $IL(t)$, $\Pr(1,'U',t)$ and $\Pr(0,'D',t)$. By switching expectation and integral sign, we can get total cost until time T.

Since we don't know what is the optimal form for the real time ordering policy, we propose several ordering policy which can be characterized by two perspectives. One is whether order policy is real time or not. Real time means order quantity reflects time dependent parameters. That is, $Q(t) \neq C$ where C is a constant. The other is whether the process of generating $Q(t)$ involving Kolmogorov CTMC differential equations. We call it 'optimization' or 'non-Optimization'

	Time-Independent	Real time
Non-Optimization	EOQ	EOQ-PSA & EOQ-PSA-t
	EOQD	EOQD-PSA & EOQD-PSA-t
Optimization	Q-nt	Q-t
		Q-t-K
		EOQ-PSA-t-ph & EOQD-PSA-t-ph

EOQ policy: $Q(t) = EOQ(t) = \sqrt{\frac{2K\overline{\lambda}}{h}}$.

EOQD policy: Use optimal Q derived from EOQD model under average value of fail and demand rate.

Q-nt policy: Optimal Q among all C where $Q(t) = C$.

EOQ-PSA policy: $Q(t) = EOQ_{PSA}(t) = \sqrt{\frac{2K\lambda(t)}{h}}$.

EOQ-PSA-t policy:

$$\begin{aligned} Q(t) &= EOQ_{PSA-t}(t) = \frac{Max(EOQ_{PSA}(t)) + Min(EOQ_{PSA}(t))}{2} \\ &\quad (1 - \frac{Max(EOQ_{PSA}(t)) - Min(EOQ_{PSA}(t))}{Max(EOQ_{PSA}(t)) - Min(EOQ_{PSA}(t))}\cos(2\pi t)) \end{aligned}$$

EOQD-PSA policy: Use optimal Q derived from EOQD model under the value of fail and demand rate at current time.

EOQ-PSA-t policy:

$$\begin{aligned} Q(t) &= EOQD_{PSA-t}(t) = \frac{Max(EOQD_{PSA}(t)) + Min(EOQD_{PSA}(t))}{2} \\ &\quad (1 - \frac{Max(EOQD_{PSA}(t)) - Min(EOQD_{PSA}(t))}{Max(EOQD_{PSA}(t)) - Min(EOQD_{PSA}(t))}\cos(2\pi t)) \end{aligned}$$

Q-t policy:

$$Q(t) = \arg \min_{Q(t)=\overline{Q}(1-RA\cos(2\pi(t+\alpha))} (C(T))$$

Q-t-K policy:

$$Q(t) = \arg \min_{Q(t)=\sum_{k=1}^{K} \overline{Q}(1-RA_k \cos(2\pi k(t+\alpha_k))} (C(T))$$

EOQ-PSA-t-ph policy:

$$Q(t) = \arg \min_{Q(t)=EOQ_{PSA-t}(t+\alpha)} (C(T))$$

EOQD-PSA-t-ph policy:

$$Q(t) = \arg \min_{Q(t)=EOQD_{PSA-t}(t+\alpha)} (C(T))$$

Synder shows EOQD outperforming EOQ under disruption. We won't consider those EOQ type policies except EOQ itself. Q-t-K policy should be better than Q-t policy since it allows more flexibility. But it involves more dimensions of decision space, so we won't consider it in this paper and we suspect benefit of increasing K to $K+1$ is diminishing very fast.

2 The benefit of real time ordering policy

For those policies involving optimization over Kolmogorov CTMC differential equations, it is hard to get optimal solution even if the form of $Q(t)$ is given. There are two ways to find 'optimal' solution. One is to use exhaustive lattice search. The other is to use existent nonlinear solver to get it done. Since it is obvious that when relative amplitude and Q approach to $\pm\infty$, $C(T)$ will approach ∞ too, and $Q(\alpha) = Q(\alpha+1)$, we can use unconstrained optimization approach. We choose lattice search to get all the results shown in the latter figures whenever optimization is needed. And we will talk about how to use Matlab optimization solver to get the satisfied result at appendix.

Our basic setting is $K = 31$, $p = 11$, $h = 1$, $p_r = 12$, $\overline{f_r} = 1$, $fRA = 0.9$, $fPH = 0$, $\overline{\lambda} = 100$, $dRA = 0$, $dPH = 0$. For the Q-nt policy, we compute all Q from $0.8\frac{EOQ+EOQD}{2}$ to $1.5\frac{EOQ+EOQD}{2}$ with increment $0.1\frac{EOQ+EOQD}{2}$. For Q-t policy, we let RA from 0 to 0.5 with increment 0.1, α from 0 to 0.9 with increment 0.1, $\overline{Q}$ from $0.8\frac{EOQ+EOQD}{2}$ to $1.5\frac{EOQ+EOQD}{2}$ with increment $0.1\frac{EOQ+EOQD}{2}$. For EOQD-PSA-t-ph policy, we let α from 0 to 0.9 with increment 0.1.

In the following subsections, we use two kinds of plots. One is absolute total cost under optimal control with regard to different ordering policy by changing the corresponding parameters. The other is relative percentage cost increase by using other policies instead of Q-t policy.

2.1 Changing Fixed Cost and Stockout Cost

We choose fixed cost from 1 to 101 with increment 10. From Fig.2, we can see total cost by using EOQ policy is worse than EOQD especially when fixed cost is low. That is because EOQ didn't adjust to

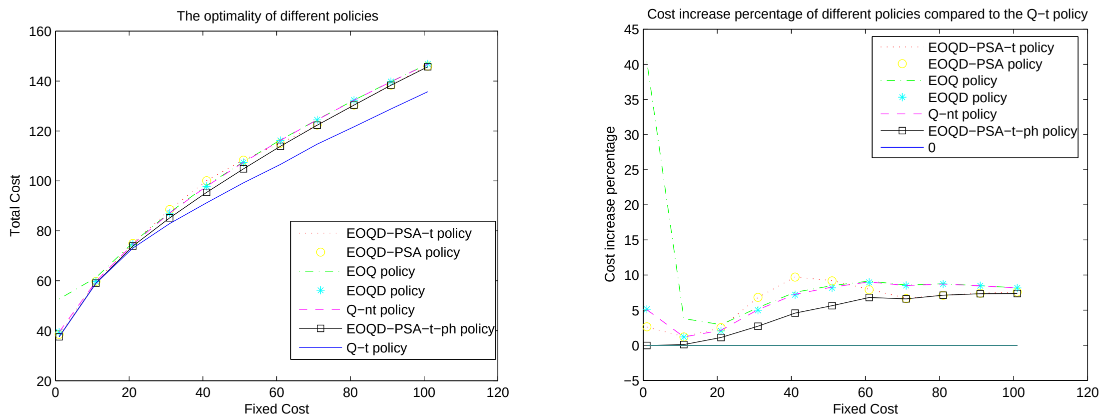

Figure 2: The total cost under different policies when fixed cost changes

stockout cost due to fixed cost. The lower fixed cost is, the more frequent ordering EOQ policy will be, so that the higher stockout cost EOQ policy will bring. When fixed cost is high enough compared to stockout cost, EOQ is close to EOQD. In our computation example, if we keep our fail rate is constant over time, the probability of supplier at down state when the retailer places its order is $1/13$, expected inspected length down period is $1/12$ due to exponential duration, and the expect demand per cycle is 100. So the total expect stockout per cycle is $100/156 \approx 0.64$. So when fixed cost is above 21, the relative ratio of fixed cost to stockout cost is above 32 where we can almost ignore the component of sotckout cost. Also from Fig.2, Q-nt policy is close to EOQD policy, that is because total cost of Q-nt policy is only affect by the average fail rate not by the fluctuation of fail rate. Since our demand is random variable, that means probability of inventory hitting zero at any time is equivalent. So EOQD is a very tight approximation of optimal Q-nt policy even if the fail rate is fluctuation.

EOQD-PSA and EOQD-PSA-t policy are worse than EOQD policy at some region. The reason is order amount under this two policies varies over time, so the probability of inventory hitting zero varies over time too. If the order quantity isn't adjusted according to the phase, it may be worse than EOQD policy sometimes. At the same time, EOQD-PSA policy is close to EOQD-PSA-t policy. From the Fig.3, we can see the their order amounts almost overlap.

EOQD-PSA-t-ph policy is close to the optimal optimal Q-t policy. The reason is that there lays a very flat canyon in the Fig.4. In this figure, the RA is fixed and for any reasonable Q, we can find a corresponding phase, such that its cost is close the minimal cost. Like EOQ model, the total cost function is really insensitive near optimal solution.

We choose stockout cost from 1 to 51 with increment 5. The result is shown in Fig.5. Opposite to the fixed cost, when stockout cost increases, EOQ policy becomes worse due to ratio of fixed cost to

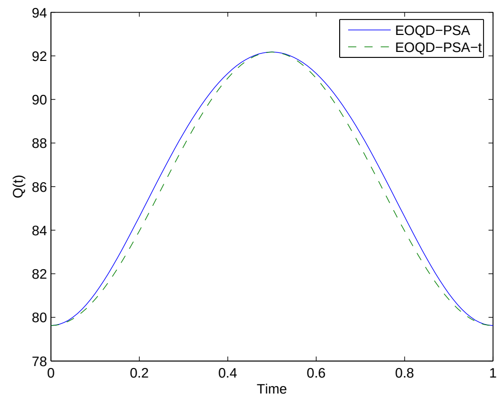

Figure 3: The relationship between EOQD-PSA and EOQD-PSA-t policy under basic setting

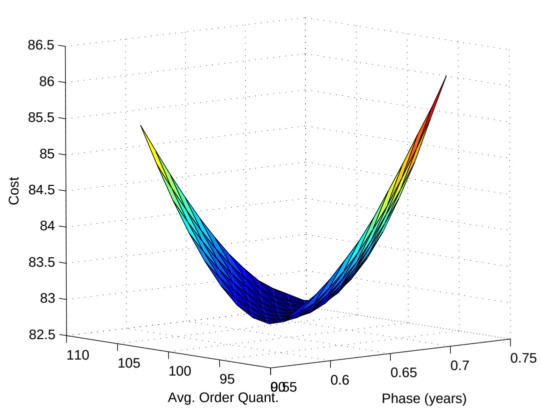

Figure 4: The cost plot when RA is fixed

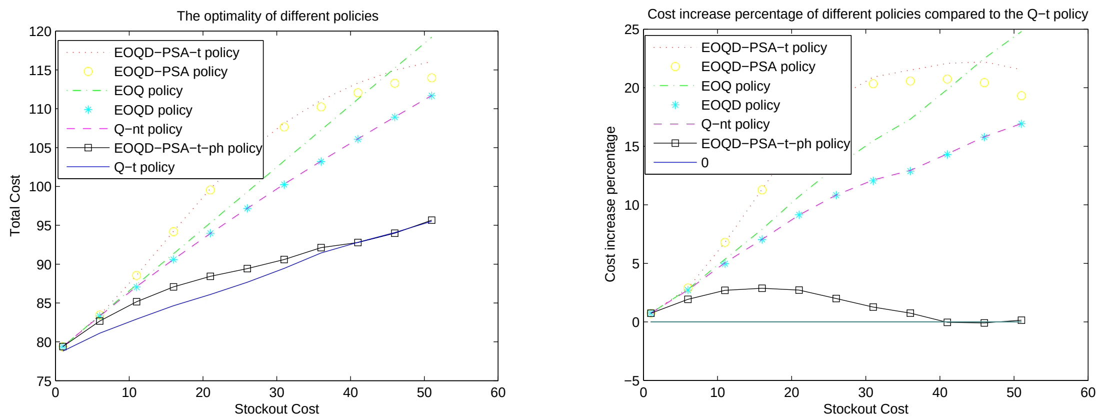

Figure 5: The total cost under different policies when stockout cost changes

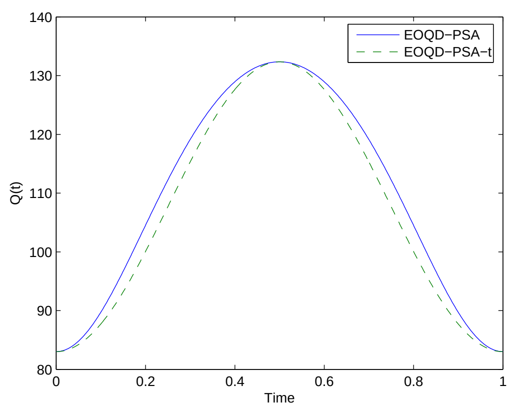

Figure 6: The relationship between EOQD-PSA and EOQD-PSA-t policy when stockout cost is 51

stockout cost decreasing. Similarly, EOQD is still close to optimal Q-nt policy. But EOQD-PSA and EOQD-PSA-t is always worse than EOQD policy. Compared to EOQD-PSA-t-ph policy, it is clear that getting right phase is critical. EOQD-PSA-t performs worse than EOQD-PSA policy when stockout cost increase. In Fig.6, the order quantity of EOQD-PSA-t policy just a little bit far away from EOQD-PSA policy compared to Fig.3. Compared to the fact that EOQD-PSA-t-ph is very close to optimal Q-t policy, we suspect that by adjusting the phase for EOQD-PSA policy directly, we can get lower cost than employing optimal Q-t policy.

The benefit of using Q-t policy increases by increasing stockout cost. When stockout is 51, the saving climbs to 15%.

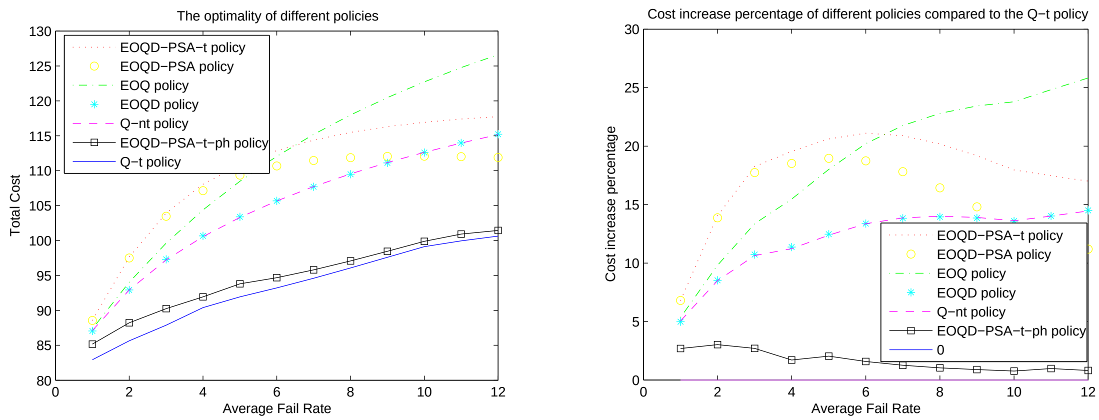

Figure 7: The total cost under different policies when average fail rate changes

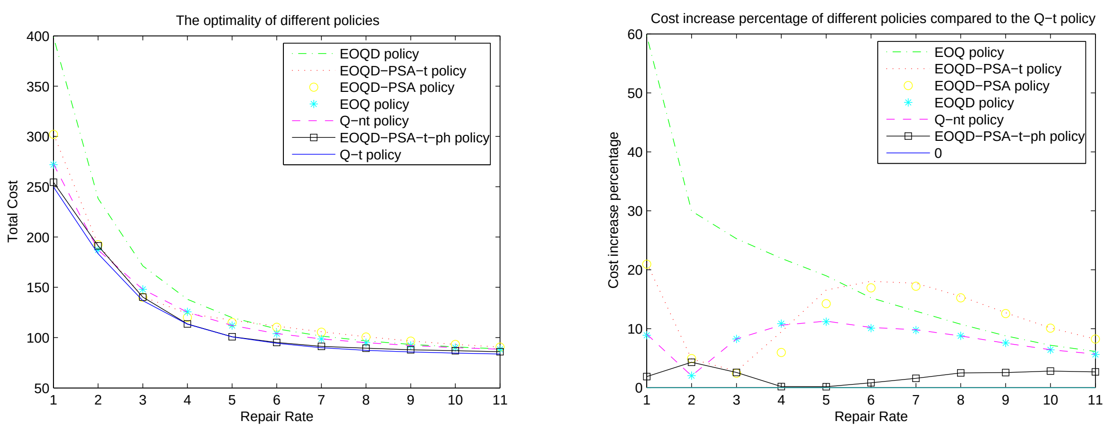

Figure 8: The total cost under different policies when repair rate changes

2.2 Changing Average Fail Rate and Repair Rate

We let average fail rate from 1 to 12 with increment 1. The result is shown in Fig.7. As we expected, EOQ becomes worse due to more frequency of disruption. EOQD-PSA and EOQD-PSA-t policy are worse than EOQD policy in the most cases. And difference between EOQD-PSA and EOQD-PSA-t increases by increasing average fail rate. Since EOQD-PSA-t-ph policy is very close optimal Q-t policy we find, we suspect using EOQD-PSA adjusted by the phase may give better result.

We let repair rate from 1 to 12 with increment 1. The result is shown in Fig.8. There is no clear patten of benefit of optimal Q-t policy. In general, the benefit of optimal Q-t policy vesus EOQD policy a little bit increase by decrease the repair rate. The spike at repair rate equal to 2 may be caused by lattic search. It should a better solution for Q-t policy than we find.

From Fig.7, we can see the total cost increase almost linear. From Fig.8, we can see the increase of total cost is not significant when repair rate decreases from 12 at the beginning. But it increases rapidly when the repair rate is more close to 0. From this experiment, we conjecture that lower frequent but serverer disruption is more cost than higher frequent but minor disruption. And suppose there is certain budget to improve the reliability of the whole system, it is interesting to learn how to allocate the resource.

2.3 Changing Relative Amplitude of Fail Rate and Demand rate

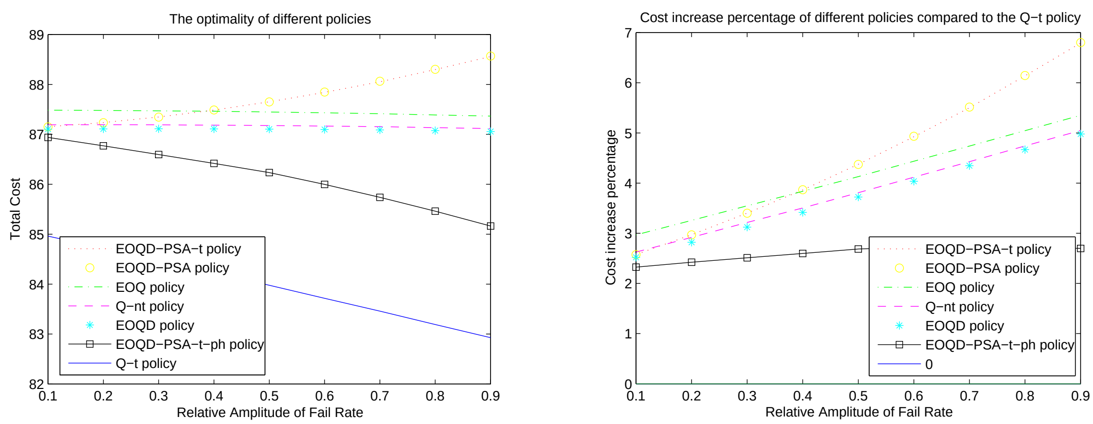

Figure 9: The total cost under different policies when relative amplitude of fail rate changes

We change the relative amplitude of both fail rate and demand rate from 0.1 to 0.9. The results are shown Fig.9 and Fig.10. As we reason in the section, the relative amplitude of fail rate shouldn't affect the total cost under any time independent ordering policy as long as the demand is constant over time. In Fig.9, the total cost of EOQ, EOQD, optimal Q-nt policy is constant. In contrast, the total cost under EOQD-PSA-t-ph policy and optimal Q-t policy is going down when fluctuation of fail rate is increase. That is because by adjusting order quantity over time, those two policies tend to let action of placing at bottom of fail rate. So the more fluctuation the fail rate is, the lower bottom of fail rate and the more benefit of Q-t policy will be.

When demand fluctuates, the cost of EOQ and EOQD isn't constant shown in the Fig.10. That's because the time of inventory level hitting zero is not equally like. When demand and fail rate has the same phase, the cost of EOQ and EOQD is increasing. If the demand phase has 0.5 lag, the cost of EOQ and EOQD may be decreasing.

It is strange that the cost of EOQD-PSA-t-ph policy also increases by the relative amplitude of demand rate, but optimal Q-t policy cause the total cost decrease at the same time.

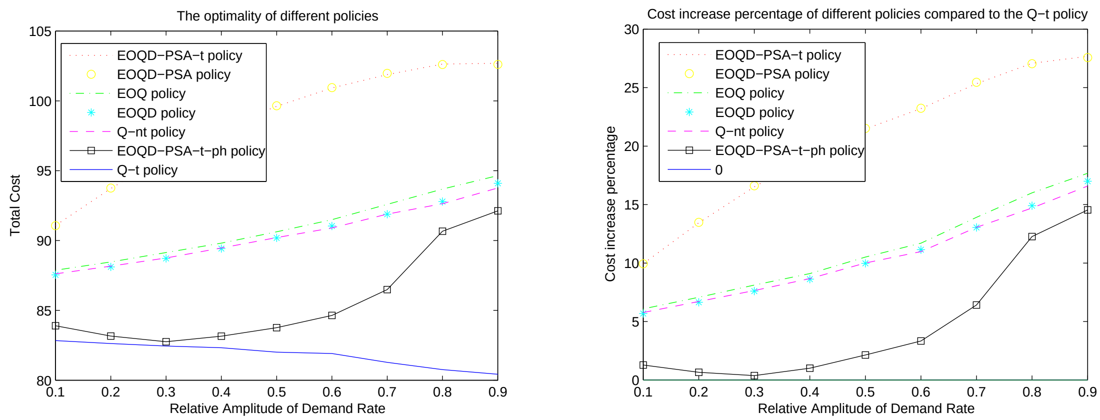

Figure 10: The total cost under different policies when relative amplitude of demand rate changes

2.4 Changing Phase of Demand Rate

We change phase of demand rate from 0 to 0.9 with increment 0.1. The result is shown in the Fig.11. From the figure, we can see the total cost under optimal Q-t policy reaches lowest point when demand phase is 0.3 lag and highest point when demand phase is 0.8. The ratio of difference to the average is around 15%. Like the relationship of sunshine with heat, the actual disruption probability should has lag compared to the fail rate. In the Fig.12, if disruption has lag, the disruption probability curve should be closer to the demand rate with 0.8. Suppose those two curve overlap wholly, that means when probability of disruption is higher, the demand rate is also higher. It causes higher stockout cost than demand phase has 0.5 lag with disruption probability.

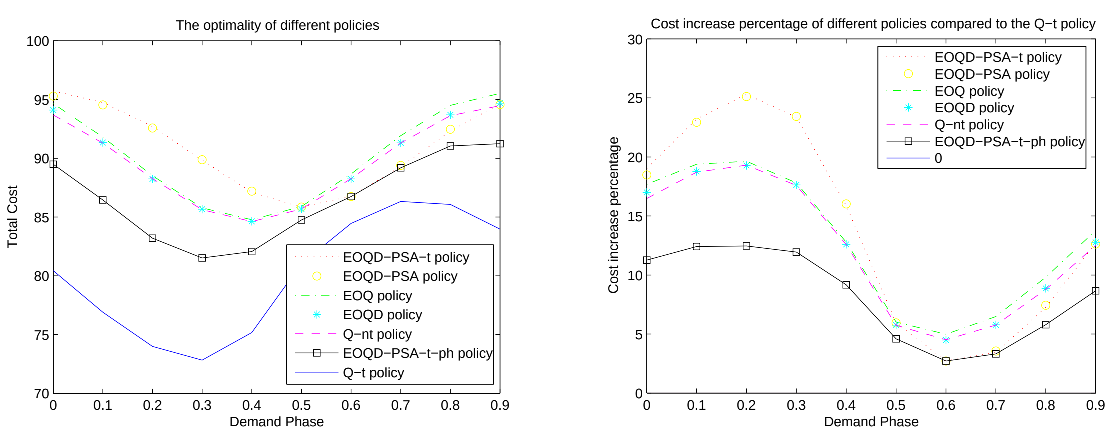

Figure 11: The total cost under different policies when phase of demand rate changes

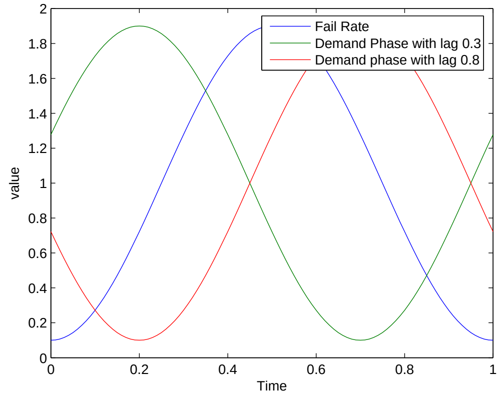

Figure 12: The plot of fail and demand rate

3 The relationship of parameters under Q-t and EOQD-PSA-t-ph policy

Our search on phase from 0 to 0.9, so 0 can represent 0 or 1 even 2. We smooth phase by adjusting with integer N. In Fig.13, we also plot average order quantity over demand mean. In the figure, we find there is a very strong linear relationship between phase and average order amount under optimal Q-t and EOQD-PSA-t-ph policies as long as demand rate is constant over time or demand and fail rate has same lag. When we do least square fit only on average order quantity over demand mean with phase, it reports

$$Phase = -0.535 + 1.15\frac{Average - order - quantity}{Demand - Mean}$$

The F statistics for this fit is 2022.46 and corresponding P-value is almost 0, which indicates we shouldn't deny that there exists linear relationship between order quantity and corresponding optimal phase. But when demand rate has the lag, it destroy such linear relationship. From the last plot of Fig.13, we can see there is no clear patten between the phase and average order amount anymore.

From the fit, we can see this relationship is almost independent on all the parameters except demand mean which is constant in our experiments. So I conjecture once we know average order quantity and mean demand, there is unique phase among $[0,)$ such that it gives us the lowest cost no matter what other parameters are. In Fig.14, I plot expected next order time if we use the optimal phase corresponding one specific average order quantity and fail rate. Next order time tends to be more concentrated on the bottom of fail rate.

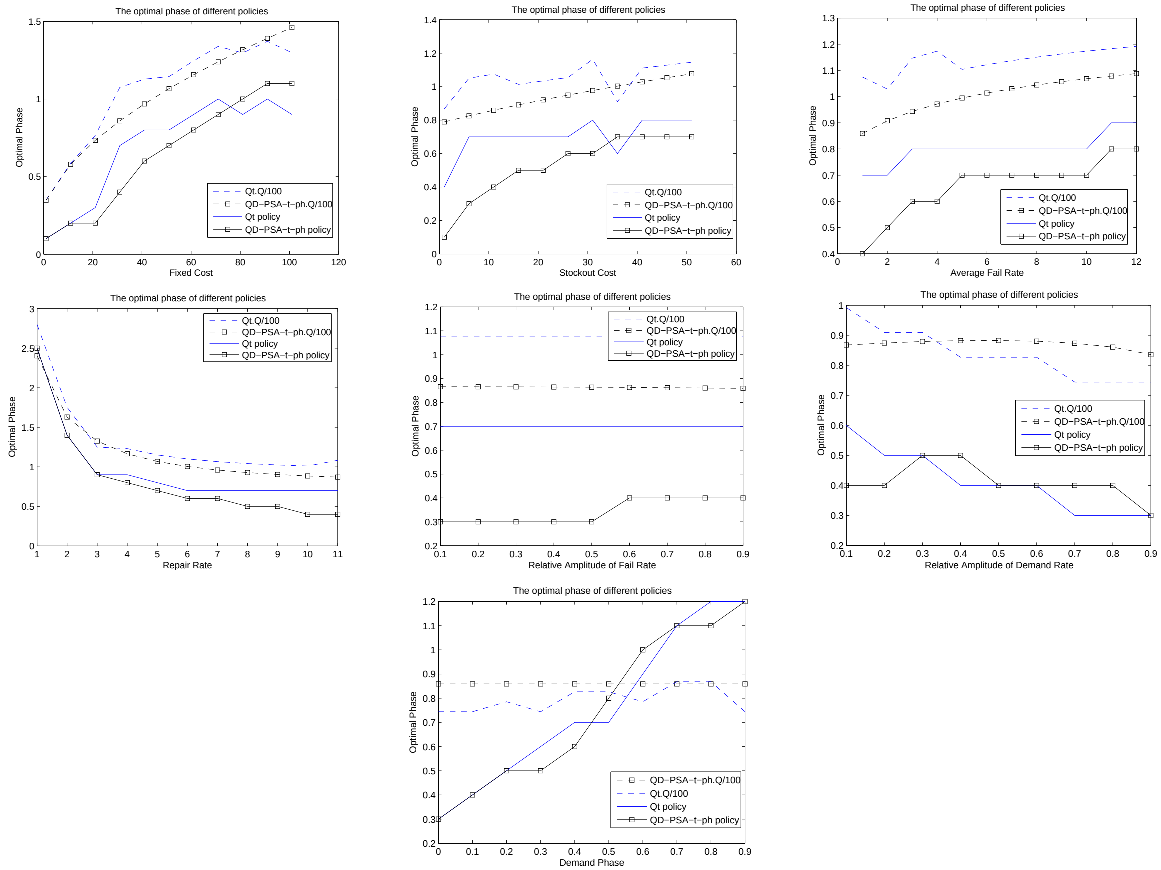

Figure 13: The relationship between phase and average order quantity under Q-t and EOQD-PSA-t-ph policy

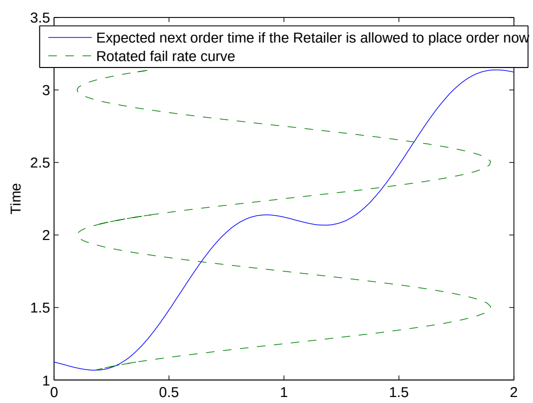

Figure 14: The relationship between expected next order time and fail rate

4 The robustness investigation of policies

Usually, the overall probability of disruption is low which means we probably have bad estimation of those probabilities needed in the CTMC. So it would be interesting to study the tradeoff between saving by employing complex control system and robust by using simple control rule. The relative amplitude and phase of fail rate are the most possible wrong estimate parameters(phase should be much easier to estimate than relative amplitude). We will use optimal Q-t and Q-nt policies under our estimations to the different combination of fail rate RA and phase. To study this problem, we use two different measures. One is worst case cost comparison and the other is average cost comparison. Here average means every combination has equally probability. This is not realistic setting. We can try to assign different probability to those combination. How to decide that distribution is another problem. We stick on the uniform distribution.

We try to change relative amplitude and phase of fail rate both from 0 to 1 with incremental 0.1. We plot the average cost for Q-nt policy. Actually as we argue in the section.2, the total cost doesn't change under any Q-nt policy even if we change relative amplitude and phase of fail rate as long as demand is constant over time. So in our experiments, highest cost, average and lowest cost curves overlaps for Q-nt policy when $dRA = 0$. There is only a little numerical difference among these three curves. We also plot highest, average and lowest cost curves. The result is shown in Fig.15.

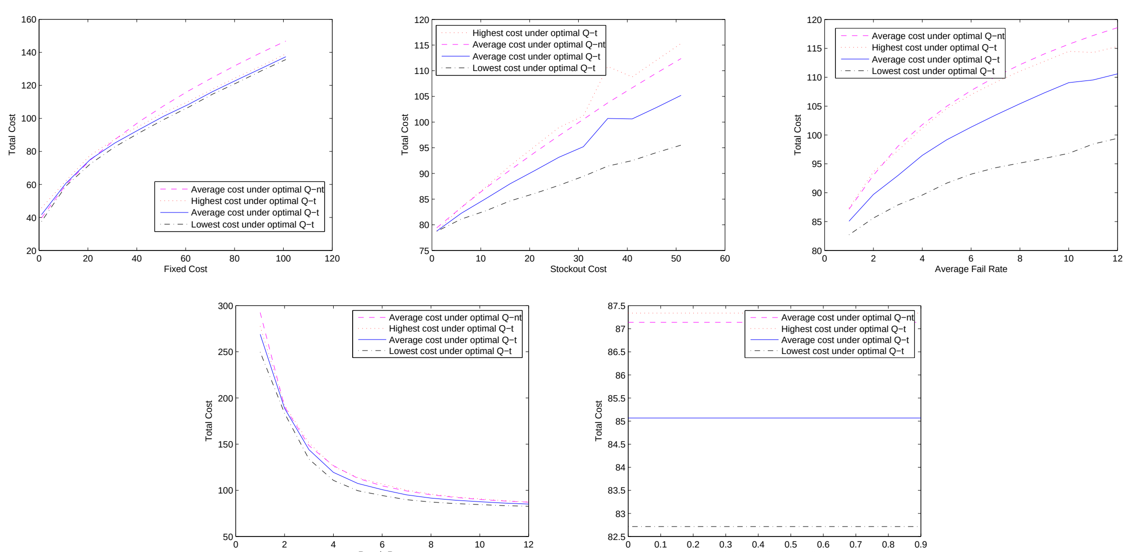

Figure 15: Robust Plot when demand rate is constant over time

When stockout cost and fail rate is low enough, there is not much difference between highest and lowest cost under Q-t policy. But in those region, the benefit is not as significant as the region. When stockout cost and fail rate is high, the wrong estimation is costly. As shown in Fig.16, the highest cost is reached when real phase has 0.5 lag to the estimation. The higher relative amplitude, the more costly the wrong estimation needs to pay for. Fortunately, based on my common sense, the higher relative amplitude, the more accurate the estimation of phase would be. Because more fluctuation will make it easy to judge the bottom and top of fail rate based on the history data. That means if we are sure about our estimation on the relative amplitude, we tend to get a accurate estimation of phase. So that we can utilize Q-t policy to achieve lower cost. If we are not sure about our estimation on the relative amplitude, we should be more conservative so that we won't lose much if we have both wrong estimation on the relative amplitude and phase.

But the last plot of Fig.15 proves my suggestion is wrong in the previous paragraph. Highest, average, lowest cost curves under Q-t policy keeps constant. So do the relative amplitude, phase and average order amount[2]. It means even if we estimate relative amplitude conservatively, it won't bring us any benefit on the robustness.

Another strange thing is that in section.2, we find EOQD-PSA-t policy sometimes is far worse than optimal Q-nt policy. But in Fig.15, the highest cost brought by the Q-t policy is close to Q-nt and most time is still better than Q-nt policy.

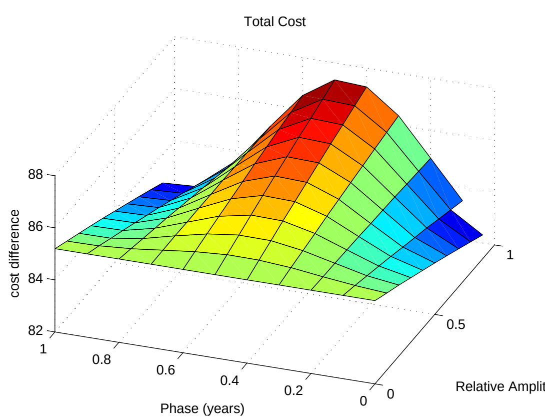

Figure 16: A typical cost plot over phase and relative amplitude when Q-t solution is fixed

When demand rate varies over time, the corresponding result is shown in Fig.17. Since the demand rate varies over time, the highest, average, lowest curve of Q-nt policy are not same. The spike in the second plot in Fig.17 are caused by lattice search which shift relative amplitude of order quantity at demand $phase = 0.5$.

[2]This result surprises me a lot, since even relative amplitude of fail rate is 0, optimal Q-nt and optimal Q-t should be the same. If not, that means even if the fail rate is constant over time, stationary policy may be worse than time varying policy

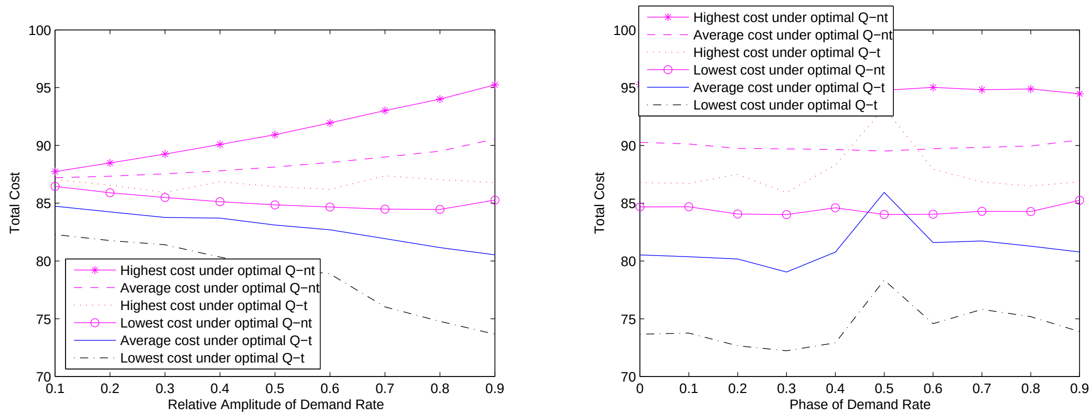

Figure 17: Robust Plot when demand rate varies

5 Conclusion

6 Appendix: How to use Matlab optimization solver to get the satisfied result

References

[1] Mahmut Parlar, David Perry, Analysis of a (Q, r, T) inventory policy with deterministic and random yields when future supply is uncertain. European Journal of Operational Research 84 (1995) 431-443

[2] Song, J.-S. and Zipkin, P. H., Inventory control with information about supply conditions, Management Science 42(10), (1996) 1409-1419

[3] Snyder, L. V. A tight approximation for a continuous-review inventory model with supplier disruptions. Working Paper

[4] Tomlin, B. T. and L. V. Snyder