Homework 1, Linear Algebra Review

Math 560

These are exercises intended to help you refresh your linear algebra knowledge for the purposes of learning Linear Programming. There will probably be more linear algebra review when we start Non-Linear Programming.

Some of these are very, very simple. I don't mean to insult your intelligence, I'm just giving you a gentle reminder of how to do basic operations. Others are a little more involved.

You may use any technology you want (but please tell me what you use). Unlike most other assignments, you may turn in this one hand-written, *and* you don't have to show any work. In fact, it's easiest if you just write your answers on a printout of this assignment.

A few words on notation:

- The apostrophe ' means "transpose".

- Some matrices are given in Matlab/Scilab notation. Spaces or commas separate entries on a single row. Semicolons denote breaks between rows. For example, [1 2 3; 4 5 6] is

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

1. Compute the following products.

    (a) Let $\vec{c} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and $\vec{x} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$. Compute $\vec{c}'\vec{x}$.

    (b) Let $\vec{c} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Compute $\vec{c}'\vec{x}$ (symbolically).

    (c) Let $\vec{c} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and $\vec{x} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$. Compute $\vec{c}\vec{x}'$. This one is not directly used in much of LP, it's just meant to be a contrast to a problem above.

    (d) Let $\vec{c} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$. Compute $\vec{c}'B^{-1}\vec{b}$.

2. For each of these problems, say how many solutions there are to the system of equations. Give at least one solution if any solutions exist, and at least two solutions if at least two solutions exist. $\vec{x}$ may change dimension from problem to problem as appropriate.

(a) $[2\ 2;\ 3\ 4]\vec{x} = [5;\ 6]$

(b) $[1\ 2\ 3;\ 4\ 5\ 6;7\ 8\ 9]\vec{x} = [10;\ 11;\ 12]$

(c) $[0\ 2\ 3;\ 4\ 5\ 6;7\ 8\ 9]\vec{x} = [10;\ 11;\ 12]$

(d) $[1\ 2\ 3;\ 4\ 5\ 6]\vec{x} = [7;\ 8]$

(e) $[1\ 2;\ 3\ 4;\ 5\ 6]\vec{x} = [7;\ 8;\ 9]$

(f) $[3\ 5;\ 21\ 35]\vec{x} = [49;\ 28]$

3. Define the following terms. If you copy from anywhere, cite your sources.

   (a) Linear combination:

   (b) Linearly dependent:

   (c) Linearly independent:

   (d) Column rank:

   (e) Row rank:

   (f) Rank:

   (g) Basis:

4. Graph the set of solutions to

   (a) $[1\ 2][x;\ y] = 10$
   (b) $[3\ 4][x;\ y] \le 12$
   (c) $[3\ 4;6\ 2][x;\ y] = [12;12]$

   Your graphs should be mostly in the first quadrant, with a little bit of the 2nd and 4th quadrants showing. This is a matter of convention in LP, rather than mathematical necessity.