Problems 7–9 are related to taxation.

7. Figure 13.13 suggests that market forces tend to drive the population to N_L. Use that figure to show how taxation or subsidization may be used to control the location of N_L. What forms might the taxation and subsidization take? (*Hint:* One cost to the fisherman is the various taxes he pays.) Apply your ideas to the whale fishery.

8. Taxation is appealing from a theoretical perspective because with a properly designed tax, desired goals can be achieved through normal market forces rather than by some artificial method (such as restricting the number of commercial permits). Assume a fish population is currently at N_b and you want to maintain it at that level by harvesting $g(N_b)$. How can a tax be determined for each fish caught to cause N_L, N_b, and N_p to coincide? (*Hint:* Consider Equation (13.8) and the condition required for $N_b = N_p$.)

9. A constant price has been assumed in all the models developed in this section. Suggest some fisheries for which that assumption is not realistic. How might you alter the assumption? How would you determine the appropriate tax?

13.4 Further Reading

Clark, Colin W. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources.* New York: Wiley, 1976.

May, Robert M., John R. Beddington, Colin W. Clark, Sidney J. Holt, & Richard M. Laws. "Management of Multispecies Fisheries." *Science* 205 (July 1979): 267–277.

APPENDIX

A Problems from the Mathematics Contest in Modeling, 1985–2008

1985: The Animal Population Problem

Choose a fish or mammal for which appropriate data are available to model it accurately. Model the animal's natural interactions with its environment by expressing population levels of different groups in terms of the significant parameters of the environment. Then adjust the model to account for harvesting in a form consistent with the actual method by which the animal is harvested. Include any outside constraints imposed by food or space limitations that are supported by the data. Consider the value of the various quantities involved, the number harvested, and the population size to devise a numerical quantity that represents the overall value of the harvest. Find a harvesting policy in terms of population size and time that optimizes the value of the harvest over a long period of time. Check that the policy optimizes this value over a realistic range of environmental conditions.

1985: The Strategic Reserve Problem

Cobalt, which is not produced in the United States, is essential to a number of industries. (Defense accounted for 17% of the cobalt production in 1979.) Most cobalt comes from central Africa, a politically unstable region. The Strategic and Critical Materials Stockpiling Act of 1946 requires a cobalt reserve that will carry the United States through a 3-year war. The government built up a cobalt stockpile in the 1950s, sold most of it in the early 1970s, and then decided to build it up again in the late 1970s, with a stockpile goal of 85.4 million pounds. About half of this stockpile had been acquired by 1982.

Build a mathematical model for managing a stockpile of the strategic metal cobalt. You will need to consider such questions as

- How big should the stockpile be?
- At what rate should it be acquired?
- What is a reasonable price to pay for the metal?

You will also want to consider such questions as

- At what point should the stockpile be drawn down?
- At what rate should it be drawn down?

- What is a reasonable price at which to sell the metal?
- How should sold metal be allocated?

Below we give more information on the sources, cost, demand, and recycling aspects of cobalt.

Useful Information on Cobalt

The government has projected a need of 25 million pounds of cobalt in 1985.

The United States has about 100 million pounds of proven cobalt deposits. Production becomes economically feasible when the price reaches $22/lb (as occurred in 1981). It takes 4 years to get operations rolling, and then 6 million pounds per year can be produced.

In 1980, 1.2 million pounds of cobalt were recycled, 7% of total consumption.

Please see Figures A.1–A.3, whose source is *Mineral Facts and Problems*, United States Bureau of Mines (Washington, DC: Government Printing Office, 1980).

Figure A.1

U.S. primary demand for cobalt, 1960–1980

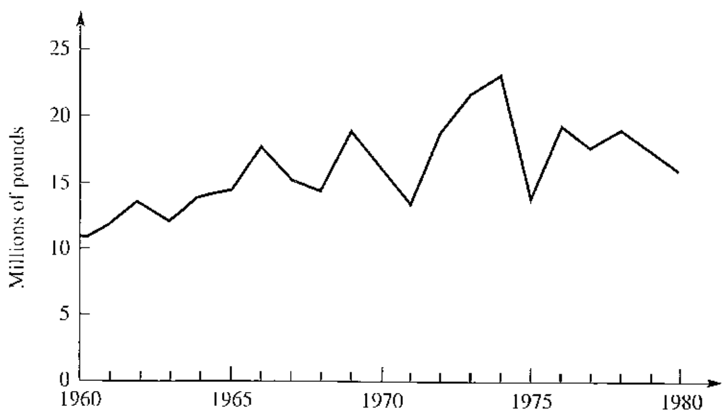
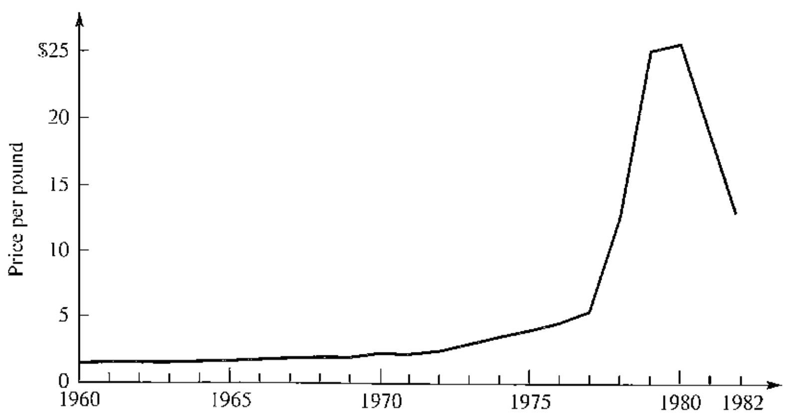


Figure A.2

Cobalt prices in the U.S. market, 1960–1982



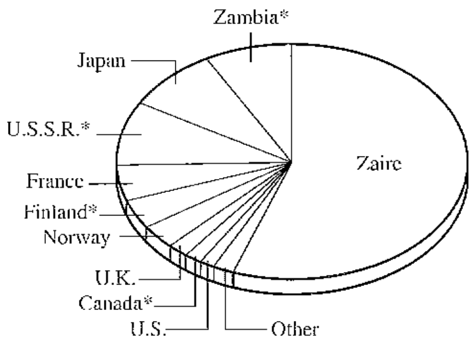


Figure A.3
Producers of refined metal and/or oxide 1979; an asterisk denotes a country with domestic production. Source: U.S. Bureau of Mines, *Mineral Facts and Problems* (1980)

1986: The Hydrographic Data Problem

The following table gives the depth Z of water in feet for surface points with rectangular coordinates X, Y in yards. The depth measurements were taken at low tide. Your ship has a draft of 5 feet. What region should you avoid within the rectangle $(150, -50) \times (200, 75)$?

X	Y	Z
129.0	7.5	4
140.0	141.5	8
108.5	28.0	6
88.0	147.0	8
185.5	22.5	6
195.0	137.5	8
105.5	85.5	8
157.5	−6.5	9
107.5	−81.0	9
77.0	3.0	8
162.0	−66.5	9
162.0	84.0	4
117.5	−38.5	9

1986: The Emergency-Facilities Location Problem

The township of Rio Rancho has hitherto not had its own emergency facilities. It has secured funds to erect two emergency facilities in 1986, each of which will combine ambulance, fire, and police services. Figure A.4 indicates the demand, or number of emergencies per square block, for 1985. The L region in the north is an obstacle, whereas the rectangle in the south is a park with a shallow pond. It takes an emergency vehicle an average of 15 seconds

Figure A.4

A map of Rio Rancho, with number of emergencies in 1985 indicated for each block

3	1	4	2	5
3	2	3	3	2
2		3	3	2
3	0		3	1
3	4	3	3	5
2	3	4	4	0
1	2	0	1	3
0	2	0	3	2
3	0		0	4
3	1	0	4	2

N

to go one block in the N–S direction and 20 seconds in the E–W direction. Your task is to locate the two facilities so as to minimize the total response time.

- Assume that the demand is concentrated at the center of the block and that the facilities will be located on corners.
- Assume that the demand is uniformly distributed on the streets bordering each block and that the facilities may be located anywhere on the streets.

1987: The Salt Storage Problem

For approximately 15 years, a midwestern state has stored salt used on roads in the winter in circular domes. Figure A.5 shows how salt has been stored in the past. The salt is brought

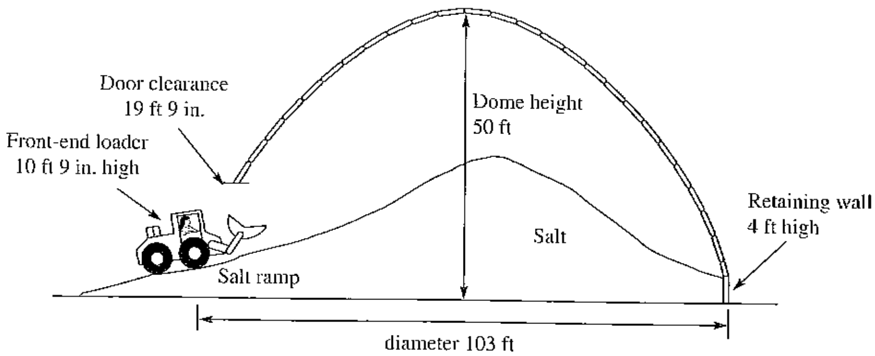


Figure A.5

Diagram of a salt storage dome

into and removed from the domes by driving front-end loaders up ramps of salt leading into the domes. The salt is piled 25–30 ft high, using the buckets on the front-end loaders.

Recently, a panel determined that this practice is unsafe. If the front-end loader got too close to the edge of the salt pile, the salt might shift, and the loader could be thrown against the retaining walls that reinforce the dome. The panel recommended that if the salt is to be piled with the use of loaders, then the piles should be restricted to a maximum height of 15 ft.

Construct a mathematical model for this situation and find a recommended maximum height for salt in the domes.

1987: The Parking Lot Problem

The owner of a paved, 100-ft-by-200-ft corner parking lot in a New England town hires you to design the layout—that is, to design how the lines are to be painted.

You realize that squeezing as many cars into the lot as possible leads to right-angle parking with the cars aligned side by side. However, inexperienced drivers have difficulty parking their cars this way, which can give rise to expensive insurance claims. To reduce the likelihood of damage to parked vehicles, the owner might then have to hire expert drivers for valet parking. On the other hand, most drivers seem to have little difficulty in parking in one attempt if there is a large enough turning radius from the access lane. Of course, the wider the access lane, the fewer cars that can be accommodated in the lot, leading to less revenue for the parking lot owner.

1988: The Railroad Flatcar Problem

Two railroad flatcars are to be loaded with seven types of packing crates. The crates have the same width and height but vary in thickness (t, in cm) and weight (w, in kg). Table A.1 gives, for each crate, the thickness, weight, and number available. Each car has 10.2 m of length available for packing the crates (like slices of toast) and can carry up to 40 metric tons. There is a special constraint on the total number of C_5, C_6, and C_7 crates because of a subsequent local trucking restriction: The total space (thickness) occupied by these crates must not exceed 302.7 cm. Load the two flatcars (Figure A.6) so as to minimize the wasted floor space.

Table A.1 The thickness, weight, and number of each kind of crate

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	
t	48.7	52.0	61.3	72.0	48.7	52.0	64.0	cm
w	2000	3000	1000	500	4000	2000	1000	kg
	8	7	9	6	6	4	8	

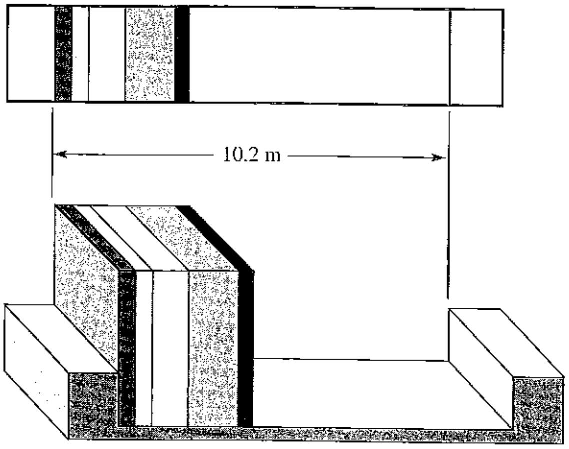


Figure A.6
Diagram of loading of a flatcar

1988: The Drug Runner Problem

Two listening posts 5.43 miles apart pick up a brief radio signal. The sensing devices were oriented at 110° and 119°, respectively, when the signal was detected (Figure A.7), and they are accurate to within 2°. The signal came from a region of active drug exchange, and it is inferred that there is a powerboat waiting for someone to pick up drugs. It is dusk, the weather is calm, and there are no currents. A small helicopter leaves a pad from Post 1 and is able to fly accurately along the 110° angle direction. The helicopter's speed is three times the speed of the boat. The helicopter will be heard when it gets within 500 ft of the boat. This helicopter has only one detection device, a searchlight. At 200 ft, it can just illuminate a circular region with a radius of 25 ft.

- Describe the (smallest) region where the pilot can expect to find the waiting boat.
- Develop an optimal search method for the helicopter.

Use a 95% confidence level in your calculations.

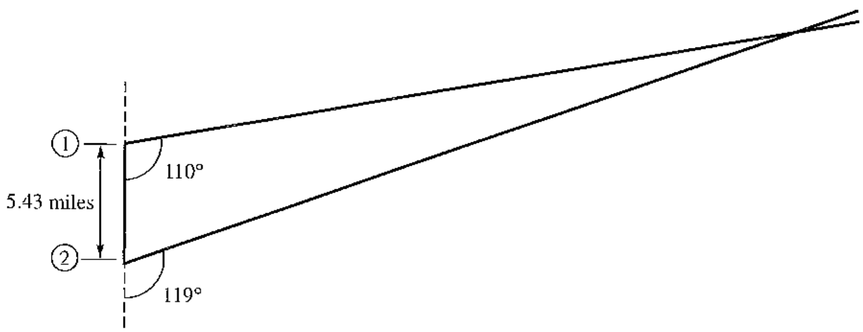


Figure A.7
Geometry of the problem

1989: The Aircraft Queuing Problem

A common procedure at airports is to assign aircraft (AC) to runways on a first-come-first-served basis. That is, as soon as an AC is ready to leave the gate (push back), the pilot calls ground control and is added to the queue. Suppose that a control tower has access to a fast online database with the following information for each AC:

- The time it is scheduled for pushback
- The time it actually pushes back
- The number of passengers on board
- The number of passengers who are scheduled to make a connection at the next stop, as well as the time to make that connection
- The schedule time of arrival at its next stop

Assume that there are seven types of AC with passenger capacities varying from 100 to 400 in steps of 50. Develop and analyze a mathematical model that takes into account both the travelers' and the airlines' satisfaction.

1989: The Midge Classification Problem

Two species of midges, Af and Apf, have been identified by biologists Grogan and Wirth (1981) on the basis of antenna and wing length (Figure A.8). Each of nine Af midges is denoted by □, and each of six Apf midges is denoted by ○. It is important to be able to classify a specimen as Af or Apf, given the antenna and wing length.

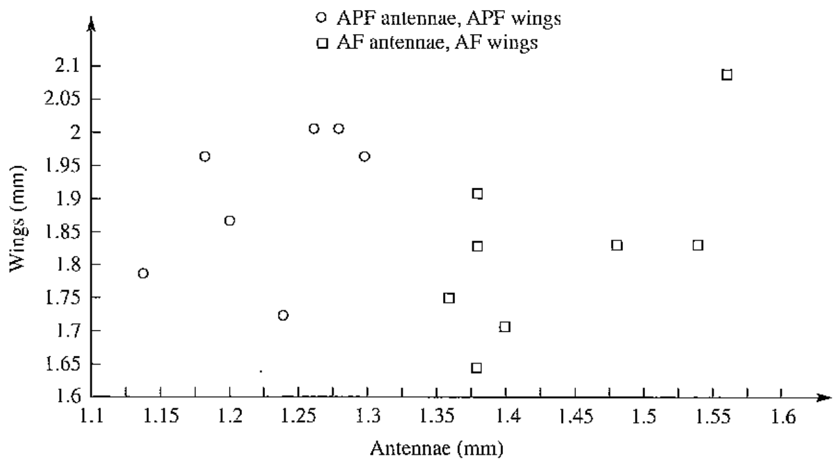


Figure A.8
Display of data collected by Grogan and Wirth (1981)

1. Given a midge that you know is species Af or Apf, how would you go about classifying it?
2. Apply your method to three specimens with (antenna, wing) lengths (1.24, 1.80), (1.28, 1.84), (1.40, 2.04).
3. Assume that species Af is a valuable pollinator and that species Apf is a carrier of a debilitating disease. Would you modify your classification scheme and if so, how?

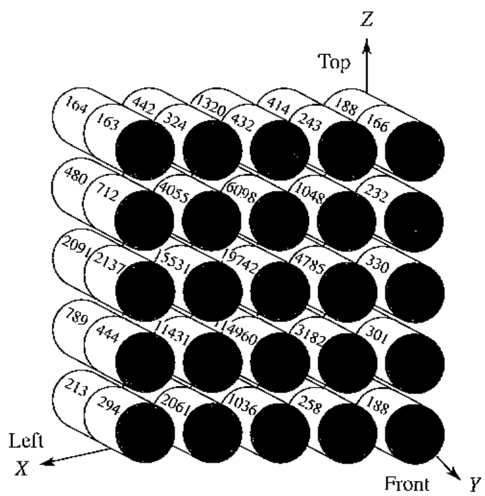
1990: The Brain–Drug Problem

Researchers on brain disorders test the effects of new medical drugs (e.g., dopamine against Parkinson's disease) with intracerebral injections. To this end, they must estimate the size and the shape of the spatial distribution of the drug after the injection to estimate accurately the region of the brain that the drug has affected.

The research data consist of the measurements of the amounts of drug in each of 50 cylindrical tissue samples (Figure A.9 and Table A.2). Each cylinder has length 0.76 mm and diameter 0.66 mm. The centers of the parallel cylinders lie on a grid with mesh $1 \times 0.76 \times 1$ mm so that the cylinders touch one another on their circular bases but not along their sides, as shown in the accompanying figure. The injection was made near the center of the cylinder with the highest scintillation count. Naturally, one expects that there is drug also between the cylinders and outside the region covered by the sample.

Figure A.9

Orientation of the cylinders of tissue



Estimate the distribution in the region affected by the drug.

One unit represents a scintillation count, or 4.753×10^{-13} mole of dopamine. For example, the table shows that the middle rear cylinder contains 28,353 units.

Table A.2 Amounts of drug in each of 50 cylindrical tissue samples

		Rear vertical section		
164	442	1,320	414	188
480	7,022	14,411	5,158	352
2,091	23,027	28,353	13,138	681
789	21,260	20,921	11,731	727
213	1,303	3,765	1,715	453
		Front vertical section		
163	324	432	243	166
712	4,055	6,098	1,048	232
2,137	15,531	19,742	4,785	330
444	11,431	14,960	3,182	301
294	2,061	1,036	258	188

1991: The Water Tank Problem

Some state water-right agencies require from communities data on the rate of water use, in gallons per hour, and the total amount of water used each day. Many communities do not have equipment to measure the flow of water into or out of the municipal tank. Instead, they can measure only the *level* of water in the tank, within 0.5% accuracy, every hour. More important, whenever the level in the tank drops below some minimum level L, a pump fills the tank up to the maximum level, H; however, there is no measurement of the pump flow either. Thus, one cannot readily relate the level in the tank to the amount of water used while the pump is working, which occurs once or twice per day, for a couple of hours each time.

Estimate the flow out of the tank $f(t)$ at all times, even when the pump is working, and estimate the total amount of water used during the day. Table A.3 gives the real data from an actual small town for one day.

The table gives the time, in seconds, since the first measurement and the level of water in the tank, in hundredths of a foot. For example, after 3316 seconds, the depth of water in

Table A.3 Water tank levels over a single day for a small town (time is in seconds and level is in 0.01 ft)

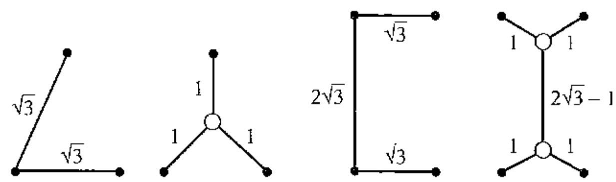
Time	Level	Time	Level	Time	Level
0	3,175	35,932	pump on	68,535	2,842
3,316	3,110	39,332	pump on	71,854	2,767
6,635	3,054	39,435	3,550	75,021	2,697
10,619	2,994	43,318	3,445	79,254	pump on
13,937	2,947	46,636	3,350	82,649	pump on
17,921	2,892	49,953	3,260	85,968	3,475
21,240	2,850	53,936	3,167	89,953	3,397
25,223	2,797	57,254	3,087	93,270	3,340
28,543	2,752	60,574	3,012		
32,284	2,697	64,554	2,927		

the tank reached 31.10 ft. The tank is a vertical circular cylinder, with a height of 40 ft and a diameter of 57 ft. Usually, the pump starts filling the tank when the level drops to about 27 ft, and the pump stops when the level rises back to about 35.50 ft.

1991: The Steiner Tree Problem

The cost for a communication line between two stations is proportional to the length of the line. The cost for conventional minimal spanning trees of a set of stations can often be cut by introducing phantom stations and then constructing a new *Steiner tree*. This device allows costs to be cut by up to 13.4% ($= 1 - \sqrt{3}/2$). Moreover, a network with n stations never requires more than $n - 2$ points to construct the cheapest Steiner tree. Two simple cases are shown in Figure A.10.

Figure A.10
Two simple cases of forming the shortest Steiner tree for a network



For local networks, it is often necessary to use rectilinear or checkerboard distances instead of straight Euclidean lines. Distances in this metric are computed as shown in Figure A.11.

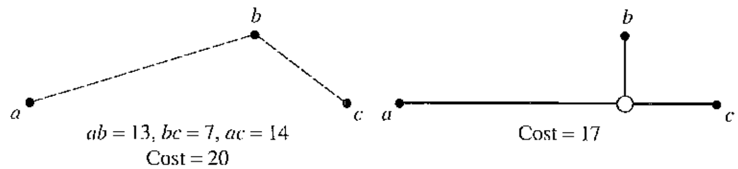


Figure A.11
Comparison of distances using straight Euclidean line distances ($ab = 13$, $bc = 7$, $ac = 14$; cost = 20) versus using rectilinear distances (cost = 17)

Suppose you wish to design a minimal-cost spanning tree for a local network with nine stations. Their rectangular coordinates are

$$a(0, 15), \quad b(5, 20), \quad c(16, 24), \quad d(20, 20), \quad e(33, 25),$$
$$f(23, 11), \quad g(35, 7), \quad h(25, 0), \quad i(10, 3)$$

You are restricted to using rectilinear lines. Moreover, all phantom stations must be located at lattice points (i.e., the coordinates must be integers). The cost for each line is its length.

1. Find a minimal-cost tree for the network.
2. Suppose each station has a cost $d^{3/2}w$, where d is the degree of the station. If $w = 1.2$, find a minimal-cost tree.
3. Try to generalize this problem.

1992: The Emergency Power-Restoration Problem

Power companies serving coastal regions must have emergency-response systems for power outages due to storms. Such systems require the input of data that allow the time and cost required for restoration to be estimated and the value of the outage judged by objective criteria. In the past, Hypothetical Electric Company (HECO) has been criticized in the media for its lack of a prioritization scheme.

You are a consultant to HECO power company. HECO possesses a computerized database with real-time access to service calls that currently require the following information:

- Time of report
- Type of requestor
- Estimated number of people affected
- Location (x, y)

Crew sites are located at coordinates (0, 0) and (40, 40), where x and y are in miles. The region serviced by HECO is within $-65 < x < 65$ and $-50 < y < 50$. The region is largely metropolitan with an excellent road network. Crews must return to their dispatch site only at the beginning and end of each shift. Company policy requires that no work be initiated until the storm leaves the area, unless the facility is a commuter railroad or hospital, which may be processed immediately if crews are available.

HECO has hired you to develop the objective criteria and schedule the work for the storm restoration requirements listed in Table A.5 using the work force described in Table A.4. Note that the first call was received at 4:20 A.M. and that the storm left the area at 6:00 A.M. Also note that many outages were not reported until much later in the day.

Table A.4 Crew descriptions

- Dispatch locations at (0, 0) and (40, 40).
- Crews consist of three trained workers.
- Crews report to the dispatch location only at the beginning and end of their shifts.
- One crew is scheduled for duty at all times on jobs assigned to each dispatch location. These crews would normally be performing routine assignments. Until the storm leaves the area, they can be dispatched for emergencies only.
- Crews work 8-hr shifts.
- There are six crew teams available at each location.
- Crews can work only one overtime shift in a work day and receive time-and-a-half for overtime.

Table A.5 Storm restoration requirements

Time (A.M.)	Location	Type	# Affected	Estimated repair time (hr for crew)
4:20	(−10, 30)	Business (cable TV)	?	6
5:30	(3, 3)	Residential	20	7
5:35	(20, 5)	Business (hospital)	240	8
5:55	(−10, 5)	Business (railroad system)	25 workers; 75,000 commuters	5
6:00	All-clear given; storm leaves area; crews can be dispatched			
6:05	(13, 30)	Residential	45	2
6:06	(5, 20)	Area	2,000	7
6:08	(60, 45)	Residential	?	9
6:09	(1, 10)	Government (city hall)	?	7
6:15	(5, 20)	Business (shopping mall)	200 workers	5
6:20	(5, −25)	Government (fire dept.)	15 workers	3
6:20	(12, 18)	Residential	350	6
6:22	(7, 10)	Area	400	12
6:25	(−1, 19)	Industry (newspaper co.)	190	10
6:40	(−20, −19)	Industry (factory)	395	7
6:55	(−1, 30)	Area	?	6
7:00	(−20, 30)	Government (high school)	1,200 students	3
7:00	(40, 20)	Government (elementary school)	1,700	?
7:00	(7, −20)	Business (restaurant)	25	12
7:00	(8, −23)	Government (police station & jail)	125	7
7:05	(25, 15)	Government (elementary school)	1,900	5
7:10	(−10, −10)	Residential	?	9
7:10	(−1, 2)	Government (college)	3,000	8
7:10	(8, −25)	Industry (computer manuf.)	450 workers	5
7:10	(18, 55)	Residential	350	10
7:20	(7, 35)	Area	400	9
7:45	(20, 0)	Residential	800	5
7:50	(−6, 30)	Business (hospital)	300	5
8:15	(0, 40)	Business (several stores)	50	6
8:20	(15, −25)	Government (traffic lights)	?	3
8:35	(−20, −35)	Business (bank)	20	5
8:50	(47, 30)	Residential	40	?
9:50	(55, 50)	Residential	?	12
10:30	(−18, −35)	Residential	10	10
10:30	(−1, 50)	Business (civic center)	150	5
10:35	(−7, −8)	Business (airport)	350 workers	4
10:50	(5, −25)	Government (fire dept.)	15	5
11:30	(8, 20)	Area	300	12

HECO has asked for a technical report for its purposes and an executive summary in lay terms that can be presented to the media. Furthermore, it would like recommendations for the future. To determine your prioritized scheduling system, you will have to make additional assumptions. Detail those assumptions. In the future, you may desire additional data. If so, detail the information desired.

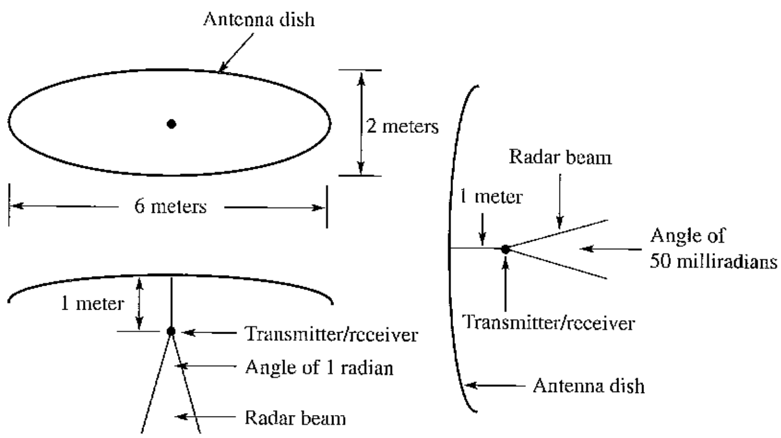
1992: The Air-Traffic-Control Radar Problem

You are to determine the power to be radiated by an air-traffic-control radar at a major metropolitan airport. The airport authority wants to minimize the power of the radar consistent with safety and cost.

The authority is constrained to operate with its existing antennae and receiver circuitry. The only option it is considering is upgrading the transmitter circuits to make the radar more powerful.

The question that you are to answer is what power (in watts) must be released by the radar to ensure detection of standard passenger aircraft at a distance of 100 km.

Figure A.12
Measurements for the radar system



Technical specifications (see also Figure A.12):

1. The radar antenna is a section of a paraboloid of revolution with focal length of 1 meter. Its projection onto a plane tangent to its vertex is an ellipse with a major axis of 6 meters and a minor axis of 2 meters. The main lobe energy beam pattern, located at the focus, is an elliptical cone that has a major axis of 1 radian and a minor axis of 50 milliradians. The antenna and beam are sketched in the figures provided.

2. The nominal class of aircraft is one that has an effective radar reflection cross section of 75 square meters. For the purposes of this problem, this means that in your initial model, the aircraft is equivalent to a 100% reflective circular disc of 75 square meters, which is centered on the axis of the antennae and is perpendicular to it. You may want to consider alternatives or refinements to this initial model.

1993: The Coal-Tipple Operations Problem

The Aspen–Boulder Coal Company runs a loading facility consisting of a large coal tipple. When the coal trains arrive, they are loaded from the tipple. The standard coal train takes 3 hours to load, and the tipple's capacity is 1.5 standard trainloads of coal. Each day, the railroad sends three standard trains to the loading facility, and they arrive at any time between 5 A.M. and 8 P.M. local time. Each of the trains has three engines. If a train arrives and sits idle while waiting to be loaded, the railroad charges a special fee, called a *demurrage*. The fee is $5000 per engine per hour. In addition, a high-capacity train arrives once a week every Thursday between 11 A.M. and 1 P.M. This special train has five engines and holds twice as much coal as a standard train. An empty tipple can be loaded directly from the mine to its capacity in 6 hours by a single loading crew. This crew (and its associated equipment) costs $9000 per hour. A second crew can be called out to increase the loading rate by conducting an additional tipple-loading operation at the cost of $12,000 per hour. Because of safety requirements, during tipple loading no trains can be loaded. Whenever train loading is interrupted to load the tipple, demurrage charges are in effect.

The management of the coal company has asked you to determine the expected annual costs of this tipple's loading operations. Your analysis should include the following considerations:

- How often should the second crew be called out?
- What are the expected monthly demurrage costs?
- If the standard trains could be scheduled to arrive at precise times, what daily schedule would minimize loading costs?
- Would a third tipple-loading crew at $12,000 per hour reduce annual operations costs?
- Can this tipple support a fourth standard train every day?

1993: The Optimal Composting Problem

An environmentally conscious institutional cafeteria is recycling customers' uneaten food into compost by means of microorganisms. Each day, the cafeteria blends the leftover food into a slurry, mixes the slurry with crisp salad wastes from the kitchen and a small amount of shredded newspaper, and feeds the resulting mixture to a culture of fungi and soil bacteria, which digest slurry, greens, and paper into usable compost. The crisp greens provide pockets of oxygen for the fungi culture, and the paper absorbs excess humidity. At times, however, the fungi culture appears unable or unwilling to digest as much of the leftovers as customers leave; the cafeteria does not blame the chef for the fungi culture's lack of appetite. Also, the cafeteria has received offers for the purchase of large quantities of its compost. Therefore, the cafeteria is investigating ways to increase its production of compost. Because it cannot yet afford to build a new composting facility, the cafeteria seeks methods to accelerate the fungi culture's activity—for instance, by optimizing the fungi culture's environment (currently held at about 120° F and 100% humidity), by optimizing the composition of the mixture fed to the fungi culture, or both.

Determine whether any relation exists between the proportions of slurry, greens, and paper in the mixture fed to the fungi culture and the rate at which the fungi culture composts the mixture. If no relation exists, state so. Otherwise, determine what proportions would accelerate the fungi culture's activity.

In addition to the technical report following the format prescribed in the contest instructions, provide a 1-page nontechnical recommendation for implementation for the cafeteria manager.

Table A.6 shows the composition of various mixtures, in pounds of each ingredient kept in separate bins, and the time it took the fungi culture to compost the mixtures, from the date fed to the date completely composted.

Table A.6 Composting data

Slurry (pounds)	Greens (pounds)	Paper (pounds)	Fed (date)	Composted (date)
86	31	0	13 Jul 90	10 Aug 90
112	79	0	17 Jul 90	13 Aug 90
71	21	0	24 Jul 90	20 Aug 90
203	82	0	27 Jul 90	22 Aug 90
79	28	0	10 Aug 90	12 Sep 90
105	52	0	13 Aug 90	18 Sep 90
121	15	0	20 Aug 90	24 Sep 90
110	32	0	22 Aug 90	8 Oct 90
82	44	9	30 Apr 91	18 Jun 91
57	60	7	2 May 91	20 Jun 91
77	51	7	7 May 91	25 Jun 91
52	38	6	10 May 91	28 Jun 91

1994: The Concrete Slab Problem

The United States Department of Housing and Urban Development (HUD) is considering constructing dwellings of various sizes, ranging from individual houses to large apartment complexes. A principal concern is to minimize recurring costs to occupants, especially the costs of heating and cooling. The region in which the construction is to take place is temperate, with a moderate variation in temperature throughout the year.

With special construction techniques, HUD engineers can build dwellings that do not need to rely on convection—that is, there is no need to rely on opening doors or windows to assist in temperature variation. The dwellings will be single-story, with concrete slab floors as the only foundation. You have been hired as a consultant to analyze the temperature variation in the concrete slab floor to determine whether the temperature averaged over the floor surface can be maintained within a prescribed comfort zone throughout the year. If so, what size/shape of slabs will permit this?

Part 1, Floor Temperature

Consider the temperature variation in a concrete slab given that the ambient temperature varies daily within the ranges given in Table A.7. Assume that the high occurs at noon and the low at midnight. Determine whether slabs can be designed to maintain a temperature averaged over the floor surface within the prescribed comfort zone, considering radiation only. Initially, assume that the heat transfer into the dwelling is through the exposed perimeter of the slab and that the top and bottom of the slabs are insulated. Comment on the appropriateness and sensitivity of these assumptions. If you cannot find a solution that satisfies Table A.7, can you find designs that satisfy a Table A.7 that you propose?

Table A.7 Daily variation in temperature

Ambient temperature		Comfort zone	
High:	85° F	High:	76° F
Low:	60° F	Low:	65° F

Part 2, Building Temperature

Analyze the practicality of the initial assumptions and extend the analysis to temperature variation within the single-story dwelling. Can the house be kept within the comfort zone?

Part 3, Cost of Construction

Suggest a design that considers HUD's objective of reducing or eliminating heating and cooling costs, considering construction restrictions and costs.

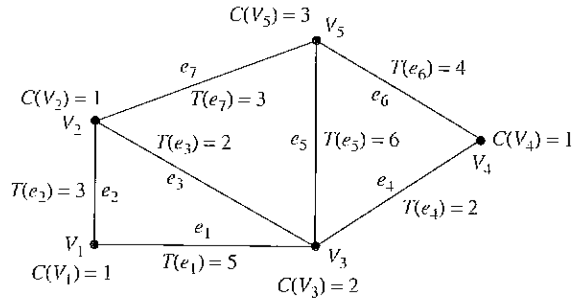
1994: The Communications Network Problem

In your company, information is shared among departments on a daily basis. This information includes the previous day's sales statistics and current production guidance. It is important to get this information out as quickly as possible.

Suppose that a communications network is to be used to transfer blocks of data (files) from one computer to another. As an example, consider the graph model in Figure A.13.

Vertices $V_1, V_2, \ldots, V_m$ represent computers, and edges $e_1, e_2, \ldots, e_n$ represent files to be transferred (between computers represented by edge endpoints). $T(e_x)$ is the time that it takes to transfer file e_x, and $C(V_y)$ is the capacity of the computer represented by V_y to transfer files simultaneously. A file transfer involves the engagement of both computers for the entire time it takes to transfer the file. For example, $C(V_y) = 1$ means that computer V_y can be involved in only one transfer at a time.

Figure A.13
Example of a file transfer network

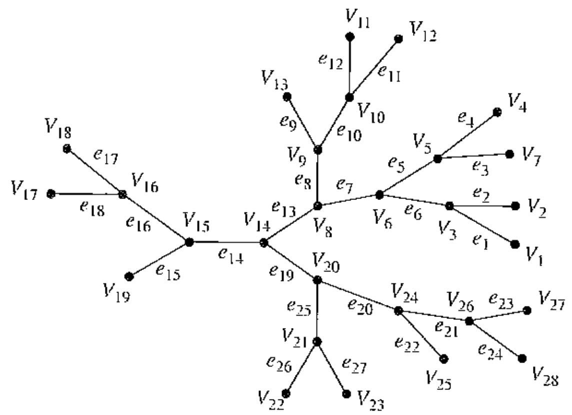


We are interested in scheduling the transfers in an optimal way, to minimize the total time that it takes to complete them all. This minimum total time is called the *makespan*. Consider the following situations for your company.

Situation A

Your corporation has 28 departments. Each department has a computer, each of which is represented by a vertex in Figure A.14. Each day, 27 files must be transferred, represented by the edges in Figure A.14. For this network, $T(e_x) = 1$ and $C(V_y) = 1$ for all x and y. Find an optimal schedule and the makespan for the given network. Can you prove to your supervisor that your makespan is the smallest possible (optimal) for the given network? Describe your approach to solving the problem. Does your approach work for the general case—that is, where $T(e_x)$, $C(V_y)$, and the graph structure are arbitrary?

Figure A.14
Network for situations A and B



Situation B

Suppose that your company changes the requirements for data transfer. You must now consider the same basic network structure (see Figure A.14) with different types and sizes of files. These files take the amount of time to transfer indicated in Table A.8 by the $T(e_x)$ terms for each edge. We still have $C(V_y) = 1$ for all y. Find an optimal schedule and the makespan for the new network. Can you prove that your makespan is the smallest possible

Table A.8 File transfer time data for situation B

x	1	2	3	4	5	6	7	8	9
$T(e_x)$	3.0	4.1	4.0	7.0	1.0	8.0	3.2	2.4	5.0
x	10	11	12	13	14	15	16	17	18
$T(e_x)$	8.0	1.0	4.4	9.0	3.2	2.1	8.0	3.6	4.5
x	19	20	21	22	23	24	25	26	27
$T(e_x)$	7.0	7.0	9.0	4.2	4.4	5.0	7.0	9.0	1.2

for the new network? Describe your approach to solving this problem. Does your approach work for the general case? Comment on any peculiar or unexpected results.

Situation C

Your corporation is considering expansion. If that happens, there are several new files (edges) that will need to be transferred daily. This expansion will also include an upgrade of the computer system. Some of the 28 departments will get new computers that can handle more than one transfer at a time. All of these changes are indicated in Figure A.15 and Tables A.9 and A.10. What is the best schedule and makespan that you can find? Can you prove that your makespan is the smallest possible for this network? Describe your approach to solving the problem. Comment on any peculiar or unexpected results.

Figure A.15
Network for situation C

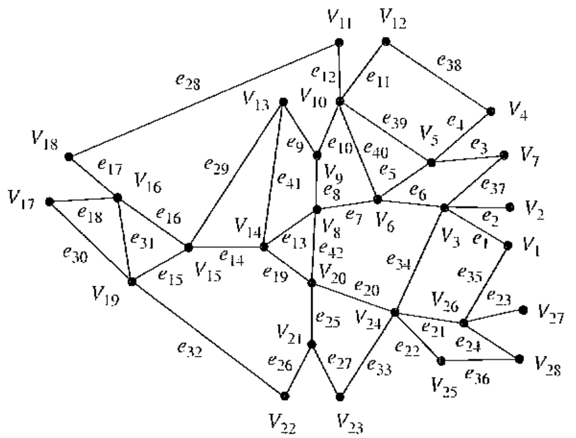


Table A.9 File transfer time data for situation C, for the added transfers

x	28	29	30	31	32	33	34	35
$T(e_x)$	6.0	1.1	5.2	4.1	4.0	7.0	2.4	9.0
x	36	37	38	39	40	41	42	
$T(e_x)$	3.7	6.3	6.6	5.1	7.1	3.0	6.1	

Table A.10 Computer capacity data for situation C

y	1	2	3	4	5	6	7	8	9	10
$C(V_y)$	2	2	1	1	1	1	1	1	2	3
y	11	12	13	14	15	16	17	18	19	
$C(V_y)$	1	1	1	2	1	2	1	1	1	
y	20	21	22	23	24	25	26	27	28	
$C(V_y)$	1	1	2	1	1	1	2	1	1	

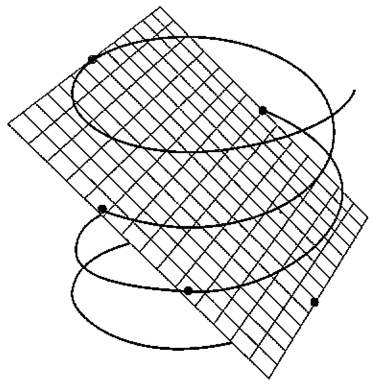
1995: The Single Helix

A small biotechnical company must design, prove, program, and test a mathematical algorithm to locate in real time all the intersections of a helix and a plane in general positions in space.

Computer-aided geometric design (CAGD) programs enable engineers to view a plane section of an object they design, such as an automobile suspension or a medical device. Engineers may also display on the plane section quantities such as air flow, stress, or temperature, coded by colors or level curves. Plane sections may be rapidly swept through the entire object to gain a three-dimensional visualization of the object and its reactions to motion, forces, or heat. To achieve such results, the computer programs must quickly and accurately locate all the intersections of the viewed plane and every part of the designed object. General equation solvers may in principle compute such intersections, but for specific problems, specific methods may prove faster and more accurate than general methods. In particular, general CAGD software may prove too slow to complete computations in real time or too large to fit in the company's finished medical devices. These considerations have led the company to the following problem.

Problem

Design, justify, program, and test a method to compute all the intersections of a plane and a helix, both in general positions (at any locations and with any orientations) in space. A segment of the helix may represent, for example, a helicoidal suspension spring or a piece of tubing in a chemical or medical apparatus.



Theoretical justification of the proposed algorithm is necessary to verify the solution from several points of view—for instance, through mathematical proofs of parts of the algorithm and through tests of the final program with known examples. Such documentation and tests will be required by government agencies for medical use.

1995: Aluacha Balaclava College

Aluacha Balaclava College, an undergraduate facility, has just hired a new Provost whose first priority is the institution of a fair and reasonable faculty compensation system. She has hired your consulting team to design a compensation system that reflects the following circumstances and principles.

Faculty are ranked as Instructor, Assistant Professor, Associate Professor, and Professor. Those with Ph.D. degrees are hired at the rank of Assistant Professor. Ph.D. candidates are hired at the rank of Instructor and promoted automatically to Assistant Professor upon completion of their degrees. Faculty may apply for promotion from Associate Professor to Professor after serving at the rank of Associate for 7 or more years. Promotions are determined by the Provost, with recommendations from a faculty committee.

Faculty salaries are for the 10-month period September through June, with raises effective beginning in September. The total amount of money available for raises varies yearly and is generally disclosed in March for the following year.

The starting salary this year for an Instructor with no prior teaching experience was \$27,000; \$32,000 for an Assistant Professor. Upon hire, faculty can receive credit for up to 7 years of teaching experience at other institutions.

Principles

1. All faculty should get a raise any year that money is available.
2. Promotion should incur a substantial benefit; e.g., promotion in the minimum possible time should result in a benefit roughly equal to 7 years of normal raises.
3. Faculty promoted after 7 or 8 years in rank with careers of at least 25 years should make roughly twice as much at retirement as a starting Ph.D.
4. Experienced faculty should be paid more than less experienced in the same rank. The effect of additional years of experience should diminish over time; that is, if two faculty stay in the same rank, their salaries should equalize over time.

Design a new pay system, first without cost-of-living increases. Incorporate cost-of-living increases, and then design a transition process for current faculty that will move all salaries toward your system without reducing anyone's salary. Existing faculty salaries, ranks, and years of service are shown in Table A.11. Discuss any refinements you think would improve your system.

The Provost requires a detailed compensation system plan for implementation, as well as a brief, clear, executive summary outlining the model, its assumptions, its strengths, its weaknesses, and its expected results, which she can present to the Board and faculty.

Table A.11 Salary data for Aluacha Balaclava College

Case	Years	Rank	Salary	Case	Years	Rank	Salary	Case	Years	Rank	Salary
1	4	ASSO	54,000	2	19	ASST	43,508	3	20	ASST	39,072
4	11	PROF	53,900	5	15	PROF	44,206	6	17	ASST	37,538
7	23	PROF	48,844	8	10	ASST	32,841	9	7	ASSO	49,981
10	20	ASSO	43,549	11	18	ASSO	42,649	12	19	PROF	60,087
13	15	ASSO	38,002	14	4	ASST	30,000	15	34	PROF	60,576
16	28	ASST	44,562	17	9	ASST	30,893	18	22	ASSO	46,351
19	21	ASSO	50,979	20	20	ASST	48,000	21	4	ASST	32,500
22	14	ASSO	38,462	23	23	PROF	53,500	24	21	ASSO	42,488
25	20	ASSO	43,892	26	5	ASST	35,330	27	19	ASSO	41,147
28	15	ASST	34,040	29	18	PROF	48,944	30	7	ASST	30,128
31	5	ASST	35,330	32	6	ASSO	35,942	33	8	PROF	57,295
34	10	ASST	36,991	35	23	PROF	60,576	36	20	ASSO	48,926
37	9	PROF	57,956	38	32	ASSO	52,214	39	15	ASST	39,259
40	22	ASSO	43,672	41	6	INST	45,500	42	5	ASSO	52,262
43	5	ASSO	57,170	44	16	ASST	36,958	45	23	ASST	37,538
46	9	PROF	58,974	47	8	PROF	49,971	48	23	PROF	62,742
49	39	ASSO	52,058	50	4	INST	26,500	51	5	ASST	33,130
52	46	PROF	59,749	53	4	ASSO	37,954	54	19	PROF	45,833
55	6	ASSO	35,270	56	6	ASSO	43,037	57	20	PROF	59,755
58	21	PROF	57,797	59	4	ASSO	53,500	60	6	ASST	32,319
61	17	ASSO	35,668	62	20	PROF	59,333	63	4	ASST	30,500
64	16	ASSO	41,352	65	15	PROF	43,264	66	20	PROF	50,935
67	6	ASST	45,365	68	6	ASSO	35,941	69	6	ASST	49,134
70	4	ASST	29,500	71	4	ASST	30,186	72	7	ASST	32,400
73	12	ASSO	44,501	74	2	ASST	31,900	75	1	ASSO	62,500
76	1	ASST	34,500	77	16	ASSO	40,637	78	4	ASSO	35,500
79	21	PROF	50,521	80	12	ASST	35,158	81	4	INST	28,500
82	16	PROF	46,930	83	24	PROF	55,811	84	6	ASST	30,128
85	16	PROF	46,090	86	5	ASST	28,570	87	19	PROF	44,612
88	17	ASST	36,313	89	6	ASST	33,479	90	14	ASSO	38,624
91	5	ASST	32,210	92	9	ASSO	48,500	93	4	ASST	35,150
94	25	PROF	50,583	95	23	PROF	60,800	96	17	ASST	38,464
97	4	ASST	39,500	98	3	ASST	52,000	99	24	PROF	56,922
100	2	PROF	78,500	101	20	PROF	52,345	102	9	ASST	35,798
103	24	ASST	43,925	104	6	ASSO	35,270	105	14	PROF	49,472
106	19	ASSO	42,215	107	12	ASST	40,427	108	10	ASST	37,021
109	18	ASSO	44,166	110	21	ASSO	46,157	111	8	ASST	32,500
112	19	ASSO	40,785	113	10	ASSO	38,698	114	5	ASST	31,170
115	1	INST	26,161	116	22	PROF	47,974	117	10	ASSO	37,793
118	7	ASST	38,117	119	26	PROF	62,370	120	20	ASSO	51,991
121	1	ASST	31,500	122	8	ASSO	35,941	123	14	ASSO	39,294
124	23	ASSO	51,991	125	1	ASST	30,000	126	15	ASST	34,638
127	20	ASSO	56,836	128	6	INST	35,451	129	10	ASST	32,756
130	14	ASST	32,922	131	12	ASSO	36,451	132	1	ASST	30,000
133	17	PROF	48,134	134	6	ASST	40,436	135	2	ASSO	54,500
136	4	ASSO	55,000	137	5	ASST	32,210	138	21	ASSO	43,160
139	2	ASST	32,000	140	7	ASST	36,300	141	9	ASSO	38,624

(*continued*)

Table A.12 *(Continued)*

Case	Years	Rank	Salary	Case	Years	Rank	Salary	Case	Years	Rank	Salary
142	21	PROF	49,687	143	22	PROF	49,972	144	7	ASSO	46,155
145	12	ASST	37,159	146	9	ASST	32,500	147	3	ASST	31,500
148	13	INST	31,276	149	6	ASST	33,378	150	19	PROF	45,780
151	5	PROF	70,500	152	27	PROF	59,327	153	9	ASSO	37,954
154	5	ASSO	36,612	155	2	ASST	29,500	156	3	PROF	66,500
157	17	ASST	36,378	158	5	ASSO	46,770	159	22	ASST	42,772
160	6	ASST	31,160	161	17	ASST	39,072	162	20	ASST	42,970
163	2	PROF	85,500	164	20	ASST	49,302	165	21	ASSO	43,054
166	21	PROF	49,948	167	5	PROF	50,810	168	19	ASSO	51,378
169	18	ASSO	41,267	170	18	ASST	42,176	171	23	PROF	51,571
172	12	PROF	46,500	173	6	ASST	35,798	174	7	ASST	42,256
175	23	ASSO	46,351	176	22	PROF	48,280	177	3	ASST	55,500
178	15	ASSO	39,265	179	4	ASST	29,500	180	21	ASSO	48,359
181	23	PROF	48,844	182	1	ASST	31,000	183	6	ASST	32,923
184	2	INST	27,700	185	16	PROF	40,748	186	24	ASSO	44,715
187	9	ASSO	37,389	188	28	PROF	51,064	189	19	INST	34,265
190	22	PROF	49,756	191	19	ASST	36,958	192	16	ASST	34,550
193	22	PROF	50,576	194	5	ASST	32,210	195	2	ASST	28,500
196	12	ASSO	41,178	197	22	PROF	53,836	198	19	ASSO	43,519
199	4	ASST	32,000	200	18	ASSO	40,089	201	23	PROF	52,403
202	21	PROF	59,234	203	22	PROF	51,898	204	26	ASSO	47,047

1996: The Submarine Detection Problem

The world's oceans contain an ambient noise field. Seismic disturbances, surface shipping, and marine mammals are sources that, in different frequency ranges, contribute to this field. We wish to consider how this ambient noise might be used to detect large moving objects (e.g., submarines located below the ocean surface). Assuming that a submarine makes no intrinsic noise, develop a method for detecting the presence of a moving submarine, its speed, its size, and its direction of travel, using only information obtained by measuring changes to the ambient noise field. Begin with noise at one fixed frequency and amplitude.

1996: The Contest Judging Problem

When determining the winner of a competition such as the Mathematical Contest in Modeling, there are generally a great many papers to judge. Let's say there are $P = 100$ papers. A group of J judges is collected to accomplish the judging. Funding for the contest constrains both the number of judges that can be obtained and the amount of time that they can judge. For example, if $P = 100$, then $J = 8$ is typical.

Ideally, each judge would read each paper and rank-order them, but there are too many papers for this. Instead, there will be a number of screening rounds in which each judge will read some number of papers and give them scores. Then some selection scheme is used to reduce the number of papers under consideration: If the papers are rank-ordered, then the bottom 30% that each judge rank-orders could be rejected. Alternatively, if the judges do not rank-order the papers, but instead give them numerical scores (e.g., from 1 to 100), then all papers falling below some cutoff level could be rejected.

The new pool of papers is then passed back to the judges, and the process is repeated. A concern is that the total number of papers that each judge reads must be substantially less than P. The process is stopped when there are only W papers left. These are the winners. Typically for $P = 100$, $W = 3$.

Your task is to determine a selection scheme, using a combination of rank-ordering, numerical scoring, and other methods, by which the final W papers will include only papers from among the best $2W$ papers. (By "best," we mean we assume that there is an absolute rank-ordering to which all judges would agree.) For example, the top three papers found by your method will consist entirely of papers from among the best six papers. Among all such methods, the one that requires each judge to read the least number of papers is desired.

Note the possibility of systematic bias in a numerical scoring scheme. For example, for a specific collection of papers, one judge could average 70 points, whereas another could average 80 points. How would you scale your scheme to accommodate changes in the content parameters (P, J, and W)?

1997: The Velociraptor Problem

The velociraptor, *Velociraptor mongoliensis*, was a predator dinosaur that lived during the late Cretaceous period approximately 75 million years ago. Paleontologists think that it was a very tenacious hunter and may have hunted in pairs or even larger packs. Unfortunately, there is no way to observe its hunting behavior in the wild as can be done with modern mammalian predators. A group of paleontologists has approached your team and asked for help in modeling the hunting behavior of the velociraptor. They hope to compare your results with field data reported by biologists studying the behaviors of lions, tigers, and similar predatory animals.

The average adult velociraptor was 3 meters long with a hip height of 0.5 meter and an approximate mass of 45 kg. It is estimated that the animal could run extremely fast, at speeds of 60 km/hr, for about 15 seconds. After that burst of speed, the animal needed to stop and recover from a buildup of lactic acid in its muscles.

Suppose that velociraptor preyed on *Thescelosaurus neglectus*, a bipedal herbivore approximately the same size as the velociraptor. A biomechanical analysis of fossilized thescelosaurus indicates that it could run at a speed of about 50 km/hr almost indefinitely.

Part 1 Assuming the velociraptor is a solitary hunter, design a mathematical model that describes a hunting strategy for a single velociraptor stalking and chasing a single prey, as well as the evasive strategy of the prey. Assume that the thescelosaurus can always detect the velociraptor when it gets within 15 meters but *may* detect this predator at even greater ranges (up to 50 meters) depending upon the nature of the habitat and weather conditions. Additionally, because of its physical structure and strength, the velociraptor has a limited turning radius when running at full speed. This radius is estimated to be three times the animal's hip height. By contrast, the thescelosaurus is extremely agile and has a turning radius of 0.5 meters.

Part 2 Assuming the more realistic situation that the velociraptor hunted in pairs, design a new model that describes a hunting strategy for two velociraptors stalking and chasing a single prey, as well as the evasive strategy of the prey. Use the same other assumptions and limitations as in Part 1.

1997: Mix Well for Fruitful Discussions

Small-group meetings are gaining popularity for the discussion of important issues, particularly long-range planning. It is believed that large groups stymie productive discussion and that a dominant personality usually controls and directs the discussion. Thus in corporation board meetings the Board will meet in small groups to discuss issues before meeting as a whole. These smaller groups still run the risk of control by a dominant personality. In an attempt to reduce this danger, it is common to schedule several sessions with a different mix of people in the groups.

A meeting of An Tostal Corporation will be attended by 29 board members of whom 9 are in-house members (i.e., employees of the corporation). The meeting is to be an all-day affair, with 3 sessions scheduled for the morning and 4 for the afternoon. The sessions will each be 45 minutes, beginning on the hour from 9:00 A.M. to 4:00 P.M., with lunch scheduled at noon. Each morning session will consist of six discussion groups, with each discussion group led by one of the corporation's six senior officers. None of these officers are board members. Thus each senior officer will lead three different discussion groups. The senior officers will not be involved in the afternoon sessions, and each of these sessions will consist of only four different discussion groups.

The president wants a list of board member assignments to discussion groups for each of the seven sessions. The assignments should achieve as much mix of the members as possible. The ideal assignment would have each board member in a discussion group with each other board member the same number of times, while minimizing common membership of groups for the different sessions.

The assignments should also satisfy the following criteria:

1. For the morning sessions, no board member should be in the same senior officer's discussion group twice.
2. No discussion group should contain a disproportionate number of in-house members.

Give a list of assignments for members 1–9 and 10–29 and officers 1–6. Indicate how well the criteria in the previous paragraphs are met. Since it is possible that some board members will cancel at the last minute or that some not scheduled will show up, an algorithm that the secretary can use to adjust the assignments with an hour's notice would be appreciated. It would be ideal if the algorithm could also be used to make assignments for future meetings involving different levels of participation for each type of attendee.

1998: MRI Scanners

Introduction

Industrial and medical diagnostic machines known as magnetic resonance imagers (MRI) scan a three-dimensional object, such as a brain, and deliver their results in the form of a three-dimensional array of pixels. Each pixel consists of one number indicating a color or a shade of gray that encodes a measure of water concentration in a small region of the scanned object at the location of the pixel. For instance, 0 can picture high water concentration in black (ventricles, blood vessels), 128 can picture a medium water concentration in gray (brain nuclei and gray matter), and 255 can picture a low water density in white (lipid-rich white matter consisting of myelinated axons). Such MRI scanners also include facilities to picture on a screen any horizontal or vertical slice through the three-dimensional array (slices are parallel to any of the three Cartesian coordinate axes). Algorithms for picturing slices through oblique planes, however, are proprietary. Current algorithms are limited in terms of the angles and parameter options available; are implemented only on heavily used dedicated workstations; lack input capabilities for marking points in the picture before slicing; and tend to blur and "feather out" sharp boundaries between the original pixels.

A more faithful, flexible algorithm implemented on a personal computer would be useful

1. for planning minimally invasive treatments,
2. for calibrating the MRI machines,
3. for investigating structures oriented obliquely in space, such as post-mortem tissue sections in animal research,
4. for enabling cross sections at any angle through a brain atlas consisting of black-and-white line drawings.

To design such an algorithm, one can access the values and locations of the pixels, but not the initial data gathered by the scanner.

Problem

Design and test an algorithm that produces sections of three-dimensional arrays by planes in any orientation in space, preserving the original gray-scale values as closely as possible.

Data Sets

The typical data set consists of a three-dimensional array A of numbers $A(i, j, k)$ that indicates the density $A(i, j, k)$ of the object at the location $(x, y, z)_{ijk}$. Typically, $A(i, j, k)$ can range from 0 through 255. In most applications, the data set is quite large.

Teams should design data sets to test and demonstrate their algorithms. The data sets should reflect conditions likely to be of diagnostic interest. Teams should also characterize data sets that limit the effectiveness of their algorithms.

Summary

The algorithm must produce a picture of the slice of the three-dimensional array by a plane in space. The plane can have any orientation and any location in space. (The plane can miss some or all data points.) The result of the algorithm should be a model of the density of the scanned object over the selected plane.

1998: Grade Inflation

Background

Some college administrators are concerned about the grading at *A Better Class* (*ABC*) College. On average, the faculty at ABC have been giving out high grades (the average grade now given out is an A−), and it is impossible to distinguish between the good and the mediocre students. The terms of a very generous scholarship allow only the top 10% of the students to be funded, so a class ranking is required.

The dean had the idea of comparing each student to the other students in each class and using this information to build up a ranking. For example, if a student obtains an A in a class in which all students obtain an A, then this student is only "average" in this class. On the other hand, if a student obtains the only A in a class, then that student is clearly "above average." Combining information from several classes might allow students to be placed in deciles (top 10%, next 10%, etc.) across the college.

Problem

Assuming that the grades given out are (A+, A, A−, B+, . . .) can the dean's idea be made to work?

Can any other schemes produce a desired ranking?

A concern is that the grade in a single class could change many student's deciles. Is this possible?

Data Sets

Teams should design data sets to test and demonstrate their algorithms. Teams should characterize data sets that limit the effectiveness of their algorithms.

1999: Deep Impact

For some time, the National Aeronautics and Space Administration (NASA) has been considering the consequences of a large asteroid impact on the earth.

As part of this effort, your team has been asked to consider the effects of such an impact were the asteroid to land in Antarctica. There are concerns that an impact there could have considerably different consequences than one striking elsewhere on the planet.

You are to assume that an asteroid is on the order of 1000 meters in diameter and that it strikes the Antarctic continent directly at the South Pole.

Your team has been asked to provide an assessment of the impact of such an asteroid. In particular, NASA would like an estimate of the probable amount and location of human casualties from this impact, an estimate of the damage done to the food production regions in the oceans of the Southern Hemisphere, and an estimate of possible coastal flooding caused by large-scale melting of the Antarctic polar ice sheet.

1999: Unlawful Assembly

Many public facilities have signs in rooms used for public gatherings which state that it is "unlawful" for the rooms to be occupied by more than a specified number of people. Presumably, this number is based on the speed with which people in the room could be evacuated via the room's exits in case of an emergency. Similarly, elevators and other facilities often have "maximum capacities" posted.

Develop a mathematical model for deciding what number to post on such a sign as being the "lawful capacity." As part of your solution, discuss criteria (other than public safety in the case of a fire or other emergency) that might govern the number of people considered "unlawful" to occupy the room (or space). Also, for the model that you construct, consider the differences between a room with movable furniture such as a cafeteria (with tables and chairs), a gymnasium, a public swimming pool, and a lecture hall with a pattern of rows and aisles. You may wish to compare and contrast what might be done for a variety of different environments: elevator, lecture hall, swimming pool, cafeteria, or gymnasium. Gatherings such as rock concerts and soccer tournaments may present special conditions.

Apply your model to one or more public facilities at your institution (or neighboring town). Compare your results with the stated capacity, if one is posted. If used, your model is likely to be challenged by parties with interests in increasing the capacity. Write an article for the local newspaper defending your analysis.

2000: Air Traffic Control

Dedicated to the memory of Dr. Robert Machol, former chief scientist of the Federal Aviation Agency

To improve safety and reduce air traffic controller workload, the Federal Aviation Agency (FAA) is considering adding, to the air traffic control system, software that would automatically detect potential aircraft flight path conflicts and alert the controller. To that end, an analyst at the FAA has posed the following problems.

Requirement A: Given two airplanes flying in space, when should the air traffic controller consider the objects to be too close and to require intervention?

Requirement B: An airspace sector is the section of three-dimensional airspace that one air traffic controller controls. Given any airspace sector, how do we measure how complex it is from an air traffic workload perspective? To what extent is complexity determined by the number of aircraft simultaneously passing through that sector

1. at any one instant?
2. during any given interval of time?
3. during a particular time of day?

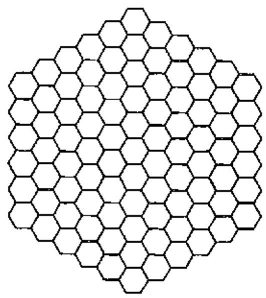
How does the number of potential conflicts arising during those periods affect complexity? Does the presence of additional software tools to automatically predict conflicts and alert the controller reduce or add to this complexity?

In addition to the guidelines for your report, write a summary (no more than two pages) that the FAA analyst can present to Jane Garvey, the FAA administrator, to defend your conclusions.

2000: Radio Channel Assignments

We seek to model the assignment of radio channels to a symmetric network of transmitter locations over a large planar area, so as to avoid interference. One basic approach is to partition the region into regular hexagons in a grid (honeycomb-style), as shown in Figure A.16, where a transmitter is located at the center of each hexagon.

Figure A.16



An interval of the frequency spectrum is to be allotted for transmitter frequencies. The interval will be divided into regularly spaced channels, which we represent by integers, 1, 2, 3, ... Each transmitter will be assigned one positive integer channel. The same channel can be used at many locations, provided that interference from nearby transmitters is avoided.

Our goal is to minimize the width of the interval in the frequency spectrum that is needed to assign channels subject to some constraints. This is achieved with the concept of a span. The span is the minimum, over all assignments satisfying the constraints, of the largest channel used at any location. It is not required that every channel smaller than the span be used in an assignment that attains the span. Let s be the length of a side of one of the hexagons. We concentrate on the case that there are two levels of interference.

Requirement A: There are several constraints on frequency assignments. First, no two transmitters within distance $4s$ of each other can be given the same channel. Second, due to spectral spreading, transmitters within distance $2s$ of each other must not be given the same or adjacent channels: Their channels must differ by at least 2. Under these constraints, what can we say about the span in Figure A.16?

Requirement B: Repeat Requirement A, assuming the grid in the example spreads arbitrarily far in all directions.

Requirement C: Repeat Requirements A and B, except now assume, more generally, that channels for transmitters within distance $2s$ differ by at least some given integer k, while those at distance at most $4s$ must still differ by at least 1. What can we say about the span and about efficient strategies for designing assignments, as a function of k?

Requirement D: Consider generalizations of the problem, such as several levels of interference or irregular transmitter placements. What other factors may be important to consider?

Requirement E: Write an article (no more than two pages) for the local newspaper explaining your findings.

2001: Choosing a Bicycle Wheel

Cyclists have different types of wheels they can use on their bicycles. The two basic types of wheels are those constructed using wire spokes and those constructed of a solid disk (Figure A.17). The spoked wheels are lighter, but the solid wheels are more aerodynamic. A solid wheel is never used on the front for a road race but can be used on the rear of the bike.

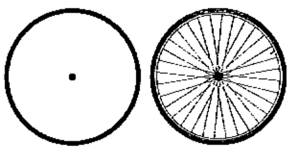


Figure A.17
A solid wheel is shown on the left and a spoked wheel is shown on the right.

Professional cyclists look at a racecourse and make an educated guess as to what kind of wheels should be used. The decision is based on the number and steepness of the hills, the weather, wind speed, the competition, and other considerations. The director sportif of your favorite team would like to have a better system in place and has asked your team for information to help determine what kind of wheel should be used for a given course.

The director sportif needs specific information to help make a decision and has asked your team to accomplish the tasks listed below. For each of the tasks, assume that the same spoked wheel will always be used on the front but there is a choice of wheels for the rear.

Task 1. Provide a table giving the wind speed at which the power required for a solid rear wheel is less than for a spoked rear wheel.

The table should include the wind speeds for different road grades ranging from 0% to 10% in 1% increments. (Road grade is defined to be the ratio of the total rise of a hill divided by the length of the road.[1]) A rider starts at the bottom of the hill at a speed of 45 kph, and the deceleration of the rider is proportional to the road grade. A rider will lose about 8 kph for a 5% grade over 100 meters.

Task 2. Provide an example of how the table could be used for a specific time trial course.

Task 3. Determine whether the table is an adequate means for deciding on the wheel configuration, and offer other suggestions as to how to make this decision.

2001: Escaping a Hurricane's Wrath

Evacuating the coast of South Carolina ahead of the predicted landfall of Hurricane Floyd in 1999 led to a monumental traffic jam. Traffic slowed to a standstill on Interstate I-26, which is the principal route going inland from Charleston to the relatively safe haven of Columbia in the center of the state. What is normally an easy 2-hour drive took up to 18 hours to complete. Many cars simply ran out of gas along the way. Fortunately, Floyd turned north and spared the state this time, but the public outcry is forcing state officials to find ways to avoid a repeat of this traffic nightmare.

The principal proposal put forth to deal with this problem is the reversal of traffic on I-26, so that both sides, including the coastal-bound lanes, have traffic headed inland from Charleston to Columbia. Plans to carry this out have been prepared (and posted on the Web) by the South Carolina Emergency Preparedness Division. Traffic reversal on principal roads leading inland from Myrtle Beach and Hilton Head is also planned.

Charleston has approximately 500,000 people, Myrtle Beach has about 200,000 people, and another 250,000 people are spread out along the rest of the coastal strip. (More accurate data, if sought, are widely available.)

The interstates have two lanes of traffic in each direction, except in metro areas, where they have three. Columbia, another metro area of around 500,000 people, does not have sufficient hotel space to accommodate the evacuees (including some coming from farther north by other routes), so some traffic continues outbound on I-26 toward Spartanburg; on I-77 north to Charlotte; and on I-20 east to Atlanta. In 1999, traffic leaving Columbia going northwest was moving very slowly.

Construct a model for the problem to investigate what strategies may reduce the congestion observed in 1999. Here are the questions that need to be addressed:

Under what conditions does the plan for turning the two coastal-bound lanes of I-26 into two lanes of Columbia-bound traffic significantly improve evacuation traffic flow?

In 1999, the simultaneous evacuation of the state's entire coastal region was ordered. Would the evacuation traffic flow improve under an alternative strategy that staggers the

[1] If the hill is viewed as a triangle, the grade is the sine of the angle at the bottom of the hill.

evacuation, perhaps county-by-county over some time period consistent with the pattern of how hurricanes affect the coast?

Several smaller highways besides I-26 extend inland from the coast. Under what conditions, would it improve evacuation flow to turn around traffic on these?

What effect would it have on evacuation flow to establish more temporary shelters in Columbia, to reduce the traffic leaving Columbia?

In 1999, many families leaving the coast brought their boats, campers, and motor homes. Many drove all of their cars. Under what conditions should there be restrictions on vehicle types or numbers of vehicles in order to guarantee timely evacuation? It has been suggested that in 1999, some of the coastal residents of Georgia and Florida, who were fleeing the earlier predicted landfalls of Hurricane Floyd to the south, came up I-95 and compounded the traffic problems. How big an impact can they have on the evacuation traffic flow?

Clearly identify what measures of performance are used to compare strategies.

Required: Prepare a short newspaper article, not to exceed two pages, explaining the results and conclusions of your study.

2002: Wind and Waterspray

An ornamental fountain in a large open plaza surrounded by buildings squirts water high into the air. On gusty days, the wind blows spray from the fountain onto passersby. The water flow from the fountain is controlled by a mechanism linked to an anemometer (which measures wind speed and direction) located on top of an adjacent building. The objective of this control is to provide passersby with an acceptable balance between an attractive spectacle and a soaking: The harder the wind blows, the lower the water volume and height to which the water is squirted, hence the less spray falls outside the pool area.

Your task is to devise an algorithm that uses data provided by the anemometer to adjust the water flow from the fountain as the wind conditions change.

2002: Airline Overbooking

You're all packed and ready to go on a trip to visit your best friend in New York City. After you check in at the ticket counter, the airline clerk announces that your flight has been overbooked. Passengers need to check in immediately to determine whether they still have a seat.

Historically, airlines know that only a certain percentage of passengers who have made reservations on a particular flight will actually take that flight. Consequently, most airlines *overbook*—that is, they take more reservations than the capacity of the aircraft. Occasionally, however, more passengers will want to take a flight than the capacity of the plane, leading to one or more passengers being *bumped* and thus unable to take the flight for which they had reservations.

Airlines deal with bumped passengers in various ways. Some are given nothing, some are booked on later flights on other airlines, and some are given some kind of cash or airline ticket incentive.

Consider the overbooking issue in light of the current situation:

Fewer flights by airlines from point A to point B

Heightened security at and around airports

Passengers' fear

Loss of billions of dollars in revenue by airlines to date

Build a mathematical model that examines the effects that different overbooking schemes have on the revenue received by an airline company in order to find an optimal overbooking strategy, i.e., the number of people by which an airline should overbook a particular flight so that the company's revenue is maximized. Ensure that your model reflects the issues above, and consider alternatives for handling "bumped" passengers. Additionally, write a short memorandum to the airline's CEO summarizing your findings and analysis.

2003: The Stunt Person

An exciting action scene in a movie is going to be filmed, and you are the stunt coordinator! A stunt person on a motorcycle will jump over an elephant and land in a pile of cardboard boxes to cushion the fall. You need to protect the stunt person, and you must also use relatively few cardboard boxes (lower cost, not seen by camera, etc.).

Your job consists of the following tasks:

- Determine what size boxes to use.
- Determine how many boxes to use.
- Determine how the boxes will be stacked.
- Determine whether any modifications to the boxes would help.
- Generalize to different combined weights (stunt person plus motorcycle) and different jump heights.

Note that in *Tomorrow Never Dies*, the James Bond character on a motorcycle jumps over a helicopter.

2003: Gamma Knife Treatment Planning

Stereotactic radiosurgery delivers a single high dose of ionizing radiation to a radiographically well-defined, small intracranial three-dimensional brain tumor without delivering any significant fraction of the prescribed dose to the surrounding brain tissue. Three modalities are commonly used in this area: the gamma knife unit, heavy charged particle beams, and external high-energy photon beams from linear accelerators.

The gamma knife unit delivers a single high dose of ionizing radiation emanating from 201 cobalt-60 unit sources through a heavy helmet. All 201 beams simultaneously intersect at the isocenter, resulting in an approximately spherical dose distribution at the effective dose levels. Irradiating the isocenter to deliver dose is termed a "shot." Shots can be represented as different spheres. Four interchangeable outer collimator helmets with beam channel diameters of 4, 8, 14, and 18 mm are available for irradiating different size volumes. For a target volume larger than one shot, multiple shots can be used to cover the entire target. In practice, most target volumes are treated with 1 to 15 shots. The target volume is a bounded, three-dimensional digital image that usually consists of millions of points.

The goal of radiosurgery is to deplete tumor cells while preserving normal structures. Because there are physical limitations and biological uncertainties involved in this therapy process, a treatment plan needs to account for all those limitations and uncertainties. In general, an optimal treatment plan is designed to meet the following requirements:

1. Minimize the dose gradient across the target volume.
2. Match specified isodose contours to the target volumes.
3. Match specified dose–volume constraints of the target and critical organ.
4. Minimize the integral dose to the entire volume of normal tissues or organs.
5. Constrain dose to specified normal tissue points below tolerance doses.
6. Minimize the maximum dose to critical volumes.

In gamma unit treatment planning, we have the following constraints:

1. Prohibit shots from protruding outside the target.
2. Prohibit shots from overlapping (to avoid hot spots).
3. Cover the target volume with effective dosage as much as possible, but at least 90% of the target volume must be covered by shots.
4. Use as few shots as possible.

Your tasks are to formulate the optimal treatment planning for a gamma knife unit as a sphere-packing problem and to propose an algorithm to find a solution. While designing your algorithm, you must keep in mind that your algorithm must be reasonably efficient.

2004: Are Fingerprints Unique?

It is a common belief that the thumbprint of every human who has ever lived is different. Develop and analyze a model that will enable you to assess the probability that this is true. Compare the odds (that you found in this problem) of misidentification by fingerprint evidence against the odds of misidentification by DNA evidence.

2004: A Faster QuickPass System

"QuickPass" systems are increasingly appearing to reduce people's time waiting in line, whether it is at tollbooths, amusement parks, or elsewhere. Consider the design of a QuickPass system for an amusement park. The amusement park has experimented by offering QuickPasses for several popular rides as a test. The idea is that for certain popular rides, you can go to a kiosk near that ride and insert your daily park entrance ticket, and out will come a slip that states that you can return to that ride at a specific time later. For example, you insert your daily park entrance ticket at 1:15 P.M., and the QuickPass states that you can come back between 3:30 and 4:30 P.M. and use your slip to enter a second, and presumably much shorter, line that will get you to the ride faster. To prevent people from obtaining QuickPasses for several rides at once, the QuickPass machines allow you to have only one active QuickPass at a time.

You have been hired as one of several competing consultants to improve the operation of QuickPass. Customers have been complaining about some anomalies in the test system. For example, customers observed that in one instance, QuickPasses were being offered for a return time as long as 4 hours later. A short time later on the same ride, the QuickPasses were given for times only an hour or so later. In some instances, the lines for people with QuickPasses are nearly as long and slow as the regular lines.

The problem, then, is to propose and test schemes for issuing QuickPasses in order to increase people's enjoyment of the amusement park. Part of the problem is to determine what criteria to use in evaluating alternative schemes. Include in your report a nontechnical summary for amusement park executives who must choose between alternatives from competing consultants.

2005: Flood Planning

Lake Murray in central South Carolina is formed by a large earthen dam, which was completed in 1930 for power production. Model the flooding downstream that would occur if there were a catastrophic earthquake that breached the dam.

Consider two particular questions:

1. Rawls Creek is a year-round stream that flows into the Saluda River a short distance downriver from the dam. How much flooding will occur in Rawls Creek from a dam failure, and how far back will it extend?
2. Could the flood be so massive downstream that water would reach up to the South Carolina State Capitol Building, which is on a hill overlooking the Congaree River?

2005: Tollbooths

Heavily traveled toll roads such as the Garden State Parkway and Interstate 95 are multilane divided highways that are interrupted at intervals by toll plazas. Because collecting tolls is usually unpopular, it is desirable to minimize motorist annoyance by limiting the amount of traffic disruption caused by the toll plazas. Commonly, a much larger number of tollbooths is provided than the number of travel lanes entering the toll plaza. Upon entering the toll plaza, the flow of vehicles fans out to the larger number of tollbooths, and when leaving the toll plaza, the flow of vehicles is required to squeeze back down to a number of travel lanes equal to the number of travel lanes before the toll plaza. Consequently, when traffic is heavy, congestion increases upon departure from the toll plaza. When traffic is very heavy, congestion also builds at the entry to the toll plaza because of the time required for each vehicle to pay the toll.

Make a model to help you determine the optimal number of tollbooths to deploy in a barrier-toll plaza. Explicitly consider the scenario where there is exactly one tollbooth per incoming travel lane. Under what conditions is this more effective than the current practice? Under what conditions is it less effective? Note that the definition of "optimal" is up to you to determine.

2006: Positioning and Moving Sprinkler Systems for Irrigation

There is a wide variety of techniques available for irrigating a field. The technologies range from advanced drip systems to periodic flooding. One approach that is used on smaller ranches is "hand move" irrigation systems. Lightweight aluminum pipes with sprinkler heads are put in place across fields, and they are moved by hand at periodic intervals to ensure that the whole field receives an adequate amount of water. This type of irrigation system is cheaper and easier to maintain than other systems. It is also flexible, allowing for use on a wide variety of fields and crops. The disadvantage is that it requires a great deal of time and effort to move and set up the equipment at regular intervals.

Given that this type of irrigation system is to be used, how can it be configured to minimize the amount of time required to irrigate a field that measures 80 meters by 30 meters? For this task you are asked to find an algorithm to determine how to irrigate the rectangular field in a way that minimizes the amount of time required by a rancher to maintain the irrigation system. One pipe set is used in the field. You should determine the number of sprinklers and the spacing between sprinklers, and you should find a schedule to move the pipes, including where to move them.

A pipe set consists of a number of pipes that can be connected together in a straight line. Each pipe has a 10-centimeter inner diameter with rotating spray nozzles that have a 0.6-centimeter inner diameter. When put together, the resulting pipe is 20 meters long. At the water source, the pressure is 420 kilopascals and has a flow rate of 150 liters per minute. No part of the field should receive more than 0.75 centimeter per hour of water, and each part of the field should receive at least 2 centimeters of water every 4 days. The total amount of water should be applied as uniformly as possible.

2006: Wheelchair Access at Airports

One of the frustrations with air travel is the need to fly through multiple airports, and each stop generally requires each traveler to change to a different airplane. This can be especially difficult for people who are not able to walk easily to a different flight's waiting area. One of the ways that an airline can make the transition easier is to provide a wheelchair and an escort to those people who ask for help. It is generally known well in advance which passengers require help, but it is not uncommon to receive notice when a passenger first registers at the airport. In rare instances, an airline may not receive notice from a passenger until just prior to landing.

Airlines are under constant pressure to keep their costs down. Wheelchairs wear out, are expensive, and require maintenance. There is also a cost for making the escorts available. Moreover, wheelchairs and their escorts must be constantly moved around the airport so that they are available to people when their flight lands. In some large airports, the time required to move across the airport is nontrivial. The wheelchairs must be stored somewhere, but space is expensive and severely limited in an airport terminal. Also, wheelchairs left in high-traffic areas represent a liability risk as people try to move around them. Finally, one of the biggest costs is the cost of holding a plane if someone must wait for an escort and becomes late for his or her flight. The latter cost is especially troubling because it can affect the airline's average flight delay, which can lead to fewer ticket sales as potential customers may choose to avoid that airline.

Epsilon Airlines planners have decided to ask a third party to help them obtain a detailed analysis of the issues and costs of keeping and maintaining wheelchairs and escorts available for passengers. The airline needs to find a way to schedule the movement of wheelchairs throughout each day in a cost-effective way. They also need to find and define the costs for budget planning in both the short and the long term.

Epsilon Airlines has asked your consultant group to put together a bid to help solve this problem. Your bid should include an overview and analysis of the situation to help the planners decide whether you fully understand their problem. They require a detailed description of an algorithm that you would like to implement which can determine where the escorts and wheelchairs should be and how they should move throughout each day. The goal is to keep the total costs as low as possible. Your bid is one of many that the airline will consider. You must make a strong case as to why your solution is the best and show that it will be able to handle a wide range of airports under a variety of circumstances.

Your bid should also include examples of how the algorithm would work for a large (at least four concourses), a medium (at least two concourses), and a small airport (one concourse) under high and low traffic loads. You should determine all potential costs and balance their respective weights. Finally, as populations begin to include a higher percentage of older people who have more time to travel but may require more aid, your report should include projections of potential costs and needs in the future, with recommendations for meeting those future needs.

2007: Gerrymandering

The United States Constitution provides that the House of Representatives shall be composed of some number of individuals (currently 435) who are elected from each state in proportion to the state's population relative to that of the country as a whole. Although this provides a way of determining how many representatives each state will have, it says nothing about how the district represented by a particular representative shall be determined geographically. This oversight has led to egregious (at least some people think so but usually not the incumbent) district shapes that look "unnatural" by some standards.

Hence the following question: Suppose you were given the opportunity to draw congressional districts for a state. How would you do so as a purely "baseline" exercise to create the "simplest" shapes for all the districts in a state? The rules insist only that each district in the state contain the same population. The definition of *simple* is up to you, but you need to make a convincing argument to voters in the state that your solution is fair. As an application of your method, draw geographically simple congressional districts for the state of New York.

2007: The Airplane Seating Problem

Airlines are free to seat passengers waiting to board an aircraft in any order whatsoever. It has become customary to seat passengers with special needs first, followed by first-class passengers (who sit at the front of the plane). Then coach and business-class passengers are seated by groups of rows, beginning with the row at the back of the plane and proceeding forward.

Apart from consideration of the passengers' wait time, from the airline's point of view time is money, and boarding time is best minimized. The plane makes money for the airline only when it is in motion, and long boarding times limit the number of trips that a plane can make in a day.

The development of larger planes, such as the Airbus A380 (800 passengers), reflects efforts to minimize boarding (and deboarding) time.

Devise and compare procedures for boarding and deboarding planes with varying numbers of passengers: small (85–210), midsize (210–330), and large (330–800).

Prepare an executive summary, not to exceed two single-spaced pages, in which you explain your conclusions to an audience of airline executives, gate agents, and flight crews.

Note: The two-page executive summary is to be included in addition to the reports required by the contest guidelines.

An article that appeared in the *New York Times* on November 14, 2006, addressed boarding and deboarding procedures currently being followed and the importance to the airline of finding better solutions. The article can be seen at http://travel2.nytimes.com/2006/11/14/business/14boarding.html.

2008: Take a Bath

Consider the effects on land from the melting of the north polar ice cap due to the predicted increase in global temperatures. Specifically, model the effects on the coast of Florida every 10 years for the next 50 years due to the melting, with particular attention given to large metropolitan areas. Propose appropriate responses to deal with this. A careful discussion of the data used is an important part of the answer.

2008: Creating Sudoku Puzzles

Develop an algorithm to construct Sudoku puzzles of varying difficulty. Develop metrics to define a difficulty level. The algorithm and metrics should be extensible to a varying number of difficulty levels. You should illustrate the algorithm with at least four difficulty levels. Your algorithm should guarantee a unique solution. Analyze the complexity of your algorithm. Your objective should be to minimize the complexity of the algorithm and meet the above requirements.

For further information on the Mathematics Contest in Modeling (MCM), write to COMAP, 57 Bedford Street, Lexington, Massachusetts 02173, or visit www.comap.com.

APPENDIX B

An Elevator Simulation Algorithm

We will define the terms used in the following algorithm and explain some of its underlying logic. Because the algorithm is complex, this approach should be more revealing than using some hypothetical numbers and taking you step by step through the algorithm. (This is a difficult program to write if you are not using GPSS or another simulation language.)

During the simulation there is a TIME clock that keeps track of the time (given in seconds). Initially, the value of TIME is 0 sec (at 7:50 A.M.), and the simulation ends when TIME reaches 4800 sec (at 9:10 A.M.). Each customer is assigned a number according to the order of his or her arrival: The first customer is labeled 1, the second customer 2, and so forth. Whenever another customer arrives at the lobby, the time between the customer's arrival and the time when the immediately preceding customer arrived is added to the TIME clock. This time between successive arrivals of customers i and $i - 1$ is labeled $between_i$ in the algorithm, and the arrival time of customer i is labeled $arrive_i$. Initially, for the customer arrival submodel, we assume that all values between 0 and 30 have an equal likelihood of occurring.

All four elevators have their own availability times, called $return_j$ for elevator j. If elevator j is currently available at the main floor, its time is the current time, so $\text{return}_j =$ TIME. If an elevator is in transit, its availability time is the time at which it will return to the main floor. Passengers enter an available elevator in the numerical order of the elevators: first elevator 1 (if it is available), next elevator 2 (if it is available), and so on. Maximum occupancy of an elevator is 12 passengers.

Whenever another customer arrives in the lobby of the building, two possible situations exist. Either an elevator is available for receiving passengers or no elevator is available and a queue is forming as customers wait for one to become available. Once an elevator becomes available, it is "tagged" for loading, and passengers can enter *only* that elevator until either it is fully occupied (with 12 passengers) or the 15-sec time delay is exceeded before the arrival of another customer. After loading, the elevator departs to deliver all of its passengers. It is assumed that, even if fully loaded with 12 passengers, the elevator waits 15 sec to load the last passenger, allow floor selection, and get under way.

To keep track of which floors have been selected during the loading period of an elevator and the number of times a particular floor has been selected, the algorithm sets up two one-dimensional arrays (having a component for each of the floors 1–12). (Although no one selects floor 1, the indexing is simplified with its inclusion.) These arrays are called $selvec_j$ and $flrvec_j$ for the tagged elevator j. If a customer selects floor 5, for instance, then a 1 is entered into the fifth component position of selvec_j and also into the