## Sample Calculus I Exam I: The Non-Calculator Part

1. Find each of the following using the unit circle. Remember to show all your work:



2. (6 points each) Find possible equations for each of the following graphs. Remember to show all work and reasoning you used to find your equations:





3. (6 points) Graphed below is the speed of the water's height as it rises in a vase, as the water is being poured into the vessel below at a constant rate (in inches<sup>3</sup>/second), and it takes exactly 10 seconds to fill it.



Sketch the vase, and explain how you came up with your picture.

4. (6 points) The distance between Cambridge and Wellesley is 10 miles. A person walks part of the way at 5 miles/hour, then jogs the rest at 8 miles/hour. Find a formula that expresses the total amount of time for the trip, T(d), as a function of d, the distance walked. Is T(d) always continuous? Explain!

## 5. If f(x) is the function

$$y = f(x) = 2(3^x) + 5$$

and g(x) is the function graphed below



find

- (a) (3 points) f(2) + g(2)
- (b) (3 points) f(g(2))

(c) (3 points) g(f(1))

(d) (3 points) Sketch the graph of the piecewise-defined function h, if h is defined by:

$$h(x) = \begin{cases} g(x) & \text{if } 0 \le x < 10\\ f(x) & \text{if } x \ge 10 \end{cases}$$

6. (20 points) The graph below is that of the function y = f(x). Use it to evaluate the limits and function values below. This is the only part of the exam where no work is required nor evaluated:



- (a)  $\lim_{x \to 2} f(x)$
- (b)  $\lim_{x \to -\infty} f(x)$
- (c)  $\lim_{x \to \infty} f(x)$
- (d)  $\lim_{x \to -4} f(x)$
- (e)  $\lim_{x \to 1} f(x)$
- (f) f(1)
- (g)  $\lim_{x \to 3^+} f(x)$
- (h)  $\lim_{x \to 2} f(x)$
- (i) List all x-values a for which  $\lim_{x \to a} f(x)$  does not exist
- (j) List all x-values a where f(x) is not continuous.

7. Evaluate each of the following limits algebraically:

(a) (6 points) 
$$\lim_{x \to 2} \frac{x^2 - 2x}{x^2 + 4x - 12}$$

(b) (6 points) 
$$\lim_{x \to 2} \frac{\frac{4}{3x} - \frac{2}{3}}{x - 2}$$

(c) (6 points) 
$$\lim_{x \to 4^-} \sqrt{\pi^x - x^3}$$

Sample Chapter 1 Exam: The Calculator Part

1. (6 points) Find as good approximation of  $\log_{30}(5)$  as possible using the graph of an exponential function.

- 2. Solve each of the following equations. If the equation can be solved algebraically, you must do so to get full points:
  - (a) (8 points)  $2 \cdot 3^{2x} = 5 \cdot 7^{3x-1}$

(b) (8 points) 
$$\left(\frac{1}{2}\right)^x = x^3 - 17x^2 + 31x - 15$$

3. (6 points each) Label each of the following functions as linear, exponential, logarithmic, trigonometric, power, polynomial, or none. You do not have to find the equations! However, your explanation should at least eliminate all of the function types you discarded and give some support for your choice of function type.

x	f(x)	g(x)	h(x)	$\ell(x)$
0	undefined	0	2.831615326	4.31
2.4	-3.923343212	18.59032006	7.758583357	21.878
4.8	-7.029626932	52.58136552	4.999999678	39.446
7.2	-8.846686424	96.59813663	2.241416895	57.014
9.6	-10.13591065	148.7225605	7.168385118	74.582
12	-11.13591065	207.8460969	6.054123552	92.15